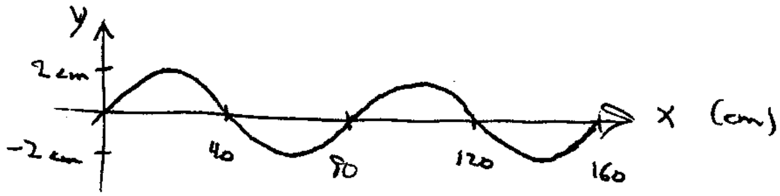
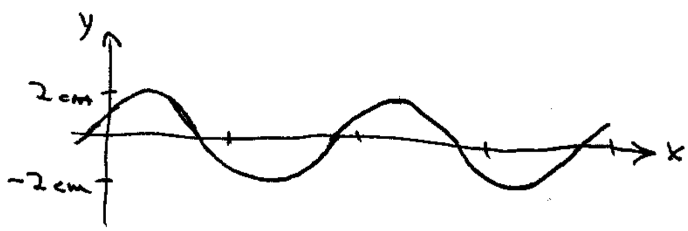
Physics 2210
(Schroeder)

Solutions to Problem Set 12

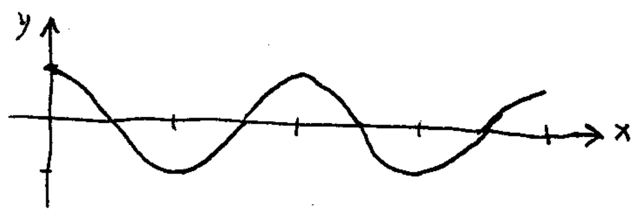
1. (a) At $t=0$, $y = 2\sin\left[2\pi \cdot \frac{x}{80}\right]$, so a full cycle is when $x = 80$:



(b) At $t = .05$ s, $y = 2\sin\left[2\pi\left(\frac{1}{8} + \frac{x}{80}\right)\right]$. Now, at $x=0$, the sine function has already gone through $\frac{1}{8}$ of a cycle.



At $t = .1$ s, $y = 2\sin\left[2\pi\left(\frac{1}{4} + \frac{x}{80}\right)\right]$



(c) From graphs, this wave is traveling in the $-x$ direction, with speed $\frac{20 \text{ cm}}{.1 \text{ s}} = \underline{200 \text{ cm/s}}$.

(d) Here $k = \frac{2\pi}{80} \text{ cm}^{-1} = .0785 \text{ cm}^{-1}$, $\omega = \frac{2\pi}{.4} \text{ s}^{-1} = 15.7 \text{ s}^{-1}$

and $v_x = -\frac{\omega}{k} = -\frac{15.7 \text{ s}^{-1}}{.0785 \text{ cm}^{-1}} = \underline{-200 \text{ cm/s}}$. ✓

2. (a) $$y = (2.0\text{ mm})\sin\left[(20\text{ m}^{-1})x - (600\text{ s}^{-1})t\right].$$

amplitude = $\underline{2.0\text{ mm}}$.

$k = 20\text{ m}^{-1}$, so $\lambda = \frac{2\pi}{k} = \frac{2\pi}{20}\text{ m} = \underline{.314\text{ m}}$.

$\omega = 600\text{ s}^{-1}$, so $f = \frac{\omega}{2\pi} = \underline{95.5\text{ s}^{-1}}$, $T = \frac{1}{f} = \underline{.0105\text{ s}}$.

$v_x = +\frac{\lambda}{T} = \underline{30\text{ m/s}}$, + because of the − inside the sine function.

(b) Transverse speed = $\frac{dy}{dt}$ for any fixed x

$$= \frac{d}{dt}\left[(2\text{ mm})\sin\left[-(600\text{ s}^{-1})t\right]\right] = (2.0\text{ mm})\cdot(-1)\cos\left[-(600\text{ s}^{-1})t\right] \times (-600\text{ s}^{-1})$$

$$= (2\text{ mm})(600\text{ s}^{-1})\cdot\underbrace{\cos\left[(600\text{ s}^{-1})t\right]}_{\max = 1}$$

$\rightarrow$ max. speed = $(2\text{ mm})(600\text{ s}^{-1}) = 1200\text{ mm/s} = \underline{1.2\text{ m/s}}$.

3. Let $v_1 = 170\text{ m/s}$, $v_2 = 180\text{ m/s}$, $\tau_1 = 120\text{ N}$, $\tau_2 = ?$

$v_1 = \sqrt{\frac{\tau_1}{\mu}}$, $v_2 = \sqrt{\frac{\tau_2}{\mu}}$,

so $v_1^2 = \frac{\tau_1}{\mu}$ $v_2^2 = \frac{\tau_2}{\mu}$ $\rightarrow \mu = \frac{\tau_1}{v_1^2} = \frac{\tau_2}{v_2^2} \rightarrow \tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2$

$\rightarrow \tau_2 = (120\text{ N})\left(\frac{180}{170}\right)^2 = \underline{135\text{ N}}$.

4. (a) speed $= \sqrt{\dfrac{\text{tension}}{\text{mass/length}}}$, independent of frequency.

So the speed is unchanged.

(b) $v_x = \dfrac{\lambda}{T} = \lambda f$, so $\lambda = \dfrac{v_x}{f}$.

Thus if f is doubled, λ is cut in half.

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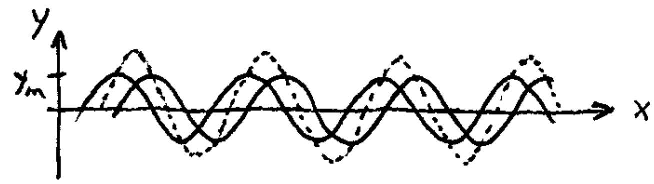
5. $v_{old} = \sqrt{\dfrac{|\vec{F}_{T,old}|}{m/L}}$, $v_{new} = \sqrt{\dfrac{|\vec{F}_{T,new}|}{m/L}}$,

so $\dfrac{v_{new}}{v_{old}} = \dfrac{\sqrt{|\vec{F}_{T,new}|/(m/L)}}{\sqrt{|\vec{F}_{T,old}|/(m/L)}} = \sqrt{\dfrac{|\vec{F}_{T,new}|}{|\vec{F}_{T,old}|}} = \sqrt{2} = \underline{1.41}$

since tension doubles

6.

I traced the solid curves from the illustration on page 402, offsetting one by 1/4 cycle:



The dashed curve is the sum of the two solid curves. Its amplitude appears to be about 1.5 × the amplitude of either solid curve: amplitude $\approx 1.5 \times y_m$.

Alternatively, from Eq. (17-42), $y_m' = 2y_m \cos\left(\frac{1}{2}\cdot\frac{\pi}{2}\right) = 2y_m \cos\frac{\pi}{4}$

$$= \frac{2}{\sqrt{2}} y_m = 1.41 \times y_m .$$

7.

(a) speed $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150\ N}{.0072\ kg/m}} = \underline{144\ m/s}$.

(b) 

$\lambda = \frac{2}{3} L = \frac{2}{3} \times 90\ cm = \underline{60\ cm}$.

(c) $f = \frac{V}{\lambda} = \frac{144\ m/s}{.6\ m} = 241\ s^{-1} = \underline{241\ Hz}$.

8.

(a) speed $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{120\ N}{.0087\ kg/1.5\ m}} = \underline{144\ m/s}$.

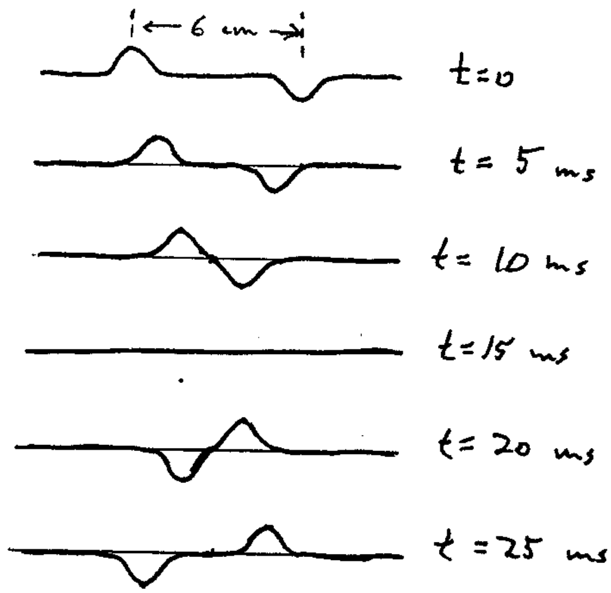
(b)  $\lambda = 2L = \underline{3\ m}$ (one loop)

 $\lambda = L = \underline{1.5\ m}$ (two loops)

(c) One loop: $f = \frac{V}{\lambda} = \frac{144\ m/s}{3\ m} = \underline{48\ Hz}$.

two loops: $f = \frac{144\ m/s}{1.5\ m} = \underline{96\ Hz}$.

9.



In 5 ms, each pulse moves $(2\ \text{m/s})(5\times10^{-3}\ \text{s}) = .01\ \text{m} = \underline{1\ \text{cm}}$, or $\frac{1}{6}$ the initial distance between them.

At t=15 ms, each pulse has reached the midpoint so they cancel each other out. Because the string is unstretched, there is no potential energy. The energy must all be kinetic, so the middle part of the string must be moving rapidly.

10.

B has units of $\frac{\text{pressure}}{\text{volume}/\text{volume}} = \text{pressure} = \frac{N}{m^2}$

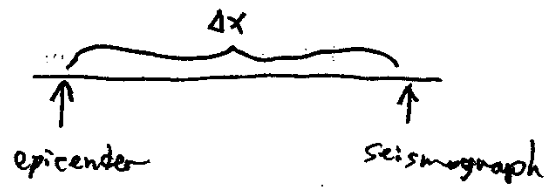
So $\sqrt{\frac{B}{\rho}}$ has units of $\sqrt{\frac{N/m^2}{kg/m^3}} = \sqrt{\frac{N\cdot m}{kg}} = \sqrt{\frac{\cancel{kg}\cdot\frac{m^2}{s^2}}{\cancel{kg}}} = \frac{m}{s}$. ✓

11. At 20°C, the speed of sound is $343\ \frac{m}{s}$.

Convert to miles: $\left(343\ \frac{m}{s}\right)\left(\frac{3.28\ ft}{m}\right)\left(\frac{1\ \text{mile}}{5280\ ft}\right) = .213$ miles/sec.

So in 5 seconds, sound travels 1.07 miles, 7% farther than indicated by this "rule". (The light from the flash travels almost instantaneously.) The time to go one km would be $\frac{1000\ m}{343\ m/s} = 2.92\ s$, so the lighting is about 1 km away for each $\underline{\text{three}}$ seconds of delay.

12.



For the P waves, $\Delta x = V_p \cdot \Delta t_p$.

for the S waves, $\Delta x = V_s \cdot \Delta t_s$

$$V_p \, \Delta t_p = V_s \, \Delta t_s$$

$$\rightarrow \frac{\Delta t_s}{\Delta t_p} = \frac{V_p}{V_s} = \frac{8 \text{ km/s}}{4.5 \text{ km/s}} = 1.78.$$

But we're told that $\Delta t_s = \Delta t_p + 3$ min.

$$\rightarrow (1.78)\Delta t_p = \Delta t_p + 3 \text{ min.} \rightarrow (.78)\Delta t_p = 3 \text{ min.} \rightarrow \Delta t_p = 3.86 \text{ min.} = 231 \text{ s.}$$

$$\rightarrow \Delta x = (8 \text{ km/s})(231 \text{ s}) = \underline{1850 \text{ km}}.$$

13.

$$v = \frac{\lambda}{T} = \lambda f \rightarrow \lambda = \frac{v}{f}.$$

Lowest pitch: $\lambda = \frac{343 \text{ m/s}}{20 \text{ s}^{-1}} = \underline{17 \text{ m}}$.

Highest pitch: $\lambda = \frac{343 \text{ m/s}}{20{,}000 \text{ s}^{-1}} \approx .017 \text{ m} = \underline{1.7 \text{ cm}}$.

14.

(a) For a pipe with both ends open, a graph of displacement looks like this:

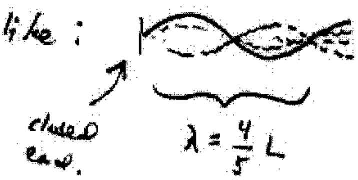
So $\lambda = 2L$. But $v_s = \lambda f$

$\rightarrow v_s = 2Lf \rightarrow L = \frac{v_s}{2f} = \frac{343\ \text{m/s}}{2 \cdot 300\ \text{s}^{-1}} = \underline{.57\ \text{m}}$.

(b) The second harmonic of this pipe looks like:

It has half the wavelength, and therefore twice the frequency, or 600 Hz.

For pipe B, the third harmonic looks like:



So $L = \frac{5}{4}\lambda = \frac{5}{4}\frac{v_s}{f} = \frac{5}{4} \cdot \frac{343\ \text{m/s}}{600\ \text{s}^{-1}}$

$= .715\ \text{m} = \underline{71.5\ \text{cm}}$.

15. The distance to the ~~top~~ farther speaker is $\sqrt{(2m)^2+(3.75m)^2}$ $= 4.25$ m.

This is $\frac{1}{2}$ m greater than the distance to the ~~lower~~ closer speaker.

(a) For minimum volume (destructive interference) $\delta = \frac{1}{2}$ m should be half a wavelength, or $\frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$, etc., so $\lambda = \frac{2\delta}{n}$, $n = 1,3,5,\ldots$

so $f = \frac{v}{\lambda} = \frac{v}{2\delta/n} = \frac{v}{2\delta}\cdot n = \left(\frac{343 \text{ m/s}}{1 \text{ m}}\right)\cdot n$, $n = 1,3,5,\ldots$

$= 343$ Hz, 1030 Hz, 1715 Hz, 2400 Hz, 3090 Hz, etc.

(b) For maximum volume (constructive interference) δ should be one full wavelength, or 2λ, 3λ, 4λ, etc., so $\lambda = \frac{\delta}{n}$, $n = 1,2,3,\ldots$

$\rightarrow f = \frac{v}{\lambda} = \frac{v}{\delta/n} = \frac{v}{\delta}\cdot n = \left(\frac{343 \text{ m/s}}{\frac{1}{2}\text{ m}}\right)\cdot n$

$= 686$ Hz, 1370 Hz, 2060 Hz, 2740 Hz, etc.