

Solutions to Problem Set 11

1. At depth h , $P = P_0 + \rho g h$ (static fluid approx.)

$$\begin{aligned} \rightarrow h &= \frac{P - P_0}{\rho g} = \frac{(60,000 \text{ atm}) - (1 \text{ atm})}{(3000 \text{ kg/m}^3)(10 \text{ N/kg})} \\ &= \frac{60,000}{30,000} \frac{\text{atm} \cdot \text{m}^3}{\text{N}} \times \left(\frac{10^5 \text{ N/m}^2}{1 \text{ atm}} \right) = 2 \times 10^5 \text{ m} \\ &= \underline{200 \text{ km}}. \end{aligned}$$

2.



At equilibrium, $|\vec{F}_b| = |\vec{F}_g|$
 \Rightarrow weight of water displaced = mg

Let $V =$ my volume, $\rho =$ my density, $\rho_w =$ density of the salt water,

then the weight of the water displaced is

$$= (\text{mass of water}) \cdot g = \rho_w \left(\frac{2}{3} V \right) \cdot g, \text{ while } m = \rho V.$$

$$\text{So } \rho_w \cdot \frac{2}{3} V \cdot g = \rho V g \Rightarrow \rho_w \cdot \frac{2}{3} = \rho \Rightarrow \rho_w = \frac{3}{2} \rho.$$

If my density is 1 g/cm^3 , then $\rho_w = 1.5 \text{ g/cm}^3$.

3. By the equation of continuity for an incompressible

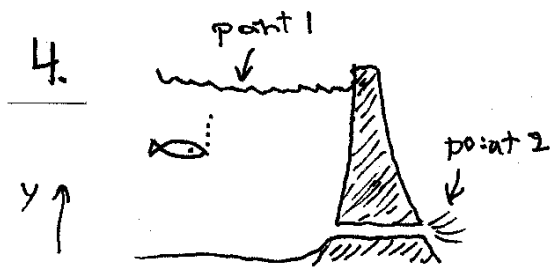
$$\text{fluid, } A_1 v_1 = A_2 v_2.$$

$$\text{Here } A_1 = \pi r^2 = \pi \left(\frac{.75 \text{ in}}{2} \right)^2 = .442 \text{ in}^2$$

$$A_2 = 24 \times \pi \left(\frac{.05 \text{ in}}{2} \right)^2 = .0471 \text{ in}^2$$

$$v_1 = 3 \text{ ft/s.}$$

$$\rightarrow v_2 = v_1 \cdot \frac{A_1}{A_2} = (3 \text{ ft/s}) \left(\frac{.442 \text{ in}^2}{.0471 \text{ in}^2} \right) = \underline{28 \text{ ft/s.}}$$



Note that both points 1 and 2 are open to the air, so $P \approx 1 \text{ atm}$ at both!

Also ρ is the same at both.

Bernoulli's equation:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$$\rightarrow \frac{1}{2} v_1^2 + g y_1 = \frac{1}{2} v_2^2 + g y_2$$

negligible

$$\rightarrow \frac{1}{2} v_2^2 = g (y_1 - y_2) \rightarrow v_2 = \sqrt{2g(y_1 - y_2)}$$

Note that this is identical to the final speed of a rock dropped from the same height. Of course! Bernoulli's equation is just another version of the law of energy conservation.

$$\begin{aligned} \text{So } v_2 &= \sqrt{2(9.8 \text{ m/s}^2)(700 \text{ ft.})\left(\frac{1 \text{ m}}{3.28 \text{ ft.}}\right)} \\ &= \underline{65 \text{ m/s}} \approx 140 \text{ mph.} \end{aligned}$$

5. Assumptions: Treat air as an incompressible fluid
Outdoor air is at atmospheric pressure at a location where it's not moving.

Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2 \quad (y_1 = y_2)$$

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where it's moving                      where it's not moving

$$\rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_2 \quad (V_2 = 0)$$

$$\rightarrow P_2 - P_1 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}(1.23 \frac{\text{kg}}{\text{m}^3})(30 \frac{\text{m}}{\text{s}})^2 = 554 \text{ N/m}^2.$$

This would be the difference in pressure between inside and outside. So the net force would be

$$|\vec{F}_{\text{net}}| = (P_2 - P_1) \cdot A = (554 \frac{\text{N}}{\text{m}^2})(4\text{m})(5\text{m})$$

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big window!

$$= \underline{11,100 \text{ N}}$$

6. (a) For any mass on a spring, $T = 2\pi\sqrt{\frac{m}{k}}$.

But here m should be the total, $M+m$,

$$\text{So } T = 2\pi\sqrt{\frac{M+m}{k}} \rightarrow T^2 = 4\pi^2 \left(\frac{M+m}{k}\right)$$

$$\rightarrow \frac{T^2 k}{4\pi^2} = M+m \rightarrow M = \left(\frac{k}{4\pi^2}\right)T^2 - m \quad \checkmark$$

(b) When $M=0$, $T = .90149 \text{ s}$.

$$\text{So } m = \frac{T^2 k}{4\pi^2} = \frac{(.90149 \text{ s})^2 (605.6 \frac{\text{N}}{\text{m}})}{4\pi^2} = 12.47 \text{ kg.}$$

$$(c) M = \frac{T^2 k}{4\pi^2} - m = \frac{(2.08832 \text{ s})^2 (605.6 \frac{\text{N}}{\text{m}})}{4\pi^2} - 12.47 \text{ kg}$$

$$= 66.90 \text{ kg} - 12.47 \text{ kg} = \underline{54.4 \text{ kg}} \quad (\approx 120 \text{ lbs.})$$

7. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ So system 1's frequency is smaller by a factor of $\sqrt{2}$.

Similarly, $T \propto \sqrt{m}$, so system 1's period is greater by $\sqrt{2}$.

Total energy = $\frac{1}{2}kA^2$, same for both. (They're the same when the springs are fully stretched, therefore they're always the same!)

Max. K.E. = same for both, since total energy is the same.

But $K = \frac{1}{2}m|v|^2 \rightarrow |v| = \sqrt{\frac{2K}{m}}$, so S1's max speed is less by $\sqrt{2}$.

#8

For a pendulum, $T = 2\pi \sqrt{\frac{L}{g}}$.

$$\text{So } T_{\text{Paris}} = 2\pi \sqrt{\frac{L}{g_{\text{Paris}}}}$$

$$T_{\text{Cayenne}} = 2\pi \sqrt{\frac{L}{g_{\text{Cayenne}}}}$$

$$\text{Divide: } \frac{T_{\text{Paris}}}{T_{\text{Cayenne}}} = \sqrt{\frac{g_{\text{Cayenne}}}{g_{\text{Paris}}}} \rightarrow \frac{g_{\text{Cayenne}}}{g_{\text{Paris}}} = \left(\frac{T_{\text{Paris}}}{T_{\text{Cayenne}}}\right)^2$$

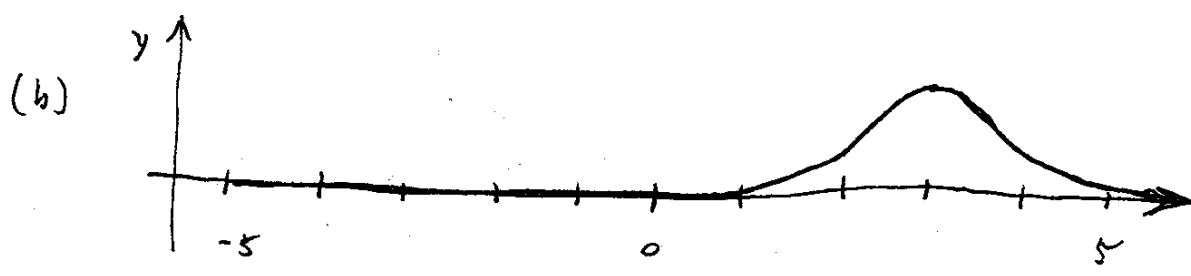
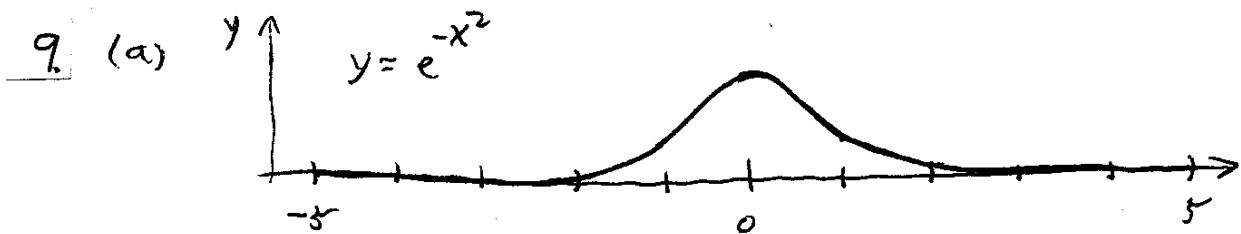
In Cayenne the clock "loses" 7.5 min./day, so T is

longer in Cayenne by 2.5 parts, in $24 \times 60 = 1440$,

$$\text{i.e., } \frac{T_{\text{Paris}}}{T_{\text{Cayenne}}} = \frac{1440}{1440 + 2.5} = .99827$$

$$\text{So } \frac{g_{\text{Cayenne}}}{g_{\text{Paris}}} = (.99827)^2 = .9965$$

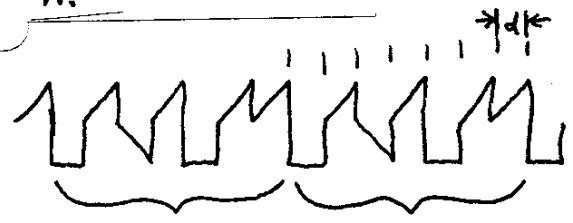
$$\rightarrow g_{\text{Cayenne}} = (.9965)(9.81 \text{ m/s}^2) = 9.776 \text{ m/s}^2 \approx \underline{9.78 \text{ m/s}^2}$$



(c) The second function is $y = e^{-(x-3)^2}$.

Now, to get the same value of y as before, you have to plug in a value of x that is larger by 3 units.

11.



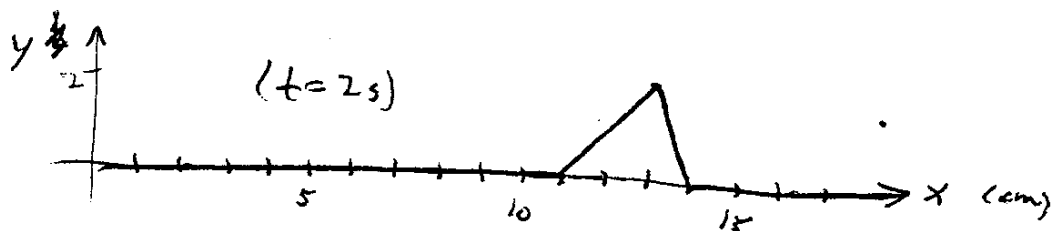
Each of these segments seems to be a full cycle, so the wavelength is $7d$.

10. (a) If $y = h(x - 5t)$, then $v_x = 5 \frac{\text{cm}}{\text{s}}$ (since x is in cm and t is in s.)

(In general, $y = f(x - v_x t)$, so here $v_x = 5$.)

(b) The pulse is moving in the $+x$ direction, because as t increases, you must plug in a larger value of x to get the same value of y .

(c) At $t = 2\text{ s}$, $y = h(x - 5t) = h(x - 10\text{ cm})$, so the graph is displaced 10 cm to the right:

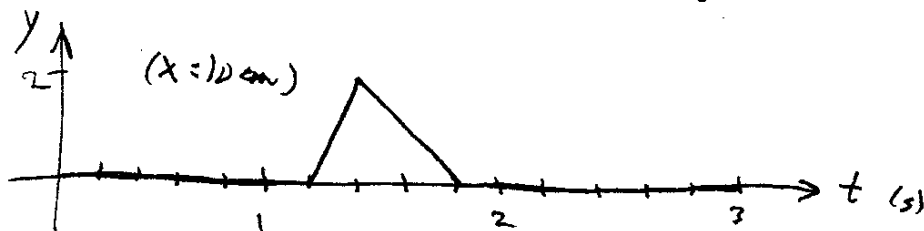


(d) At $x = 10\text{ cm}$, $y = h(10\text{ cm} - 5\frac{\text{cm}}{\text{s}} \cdot t)$.

This reaches a max. when $10 - 5t = 3$ or $5t = 7$ or $t = \frac{7}{5}\text{ s}$.

It's zero when $10 - 5t = 1$ or $t = \frac{9}{5}\text{ s}$,

and when $10 - 5t = 4$ or $t = \frac{6}{5}\text{ s}$.



12.

$$v_x = \text{"Speed"} = \frac{\lambda}{T} \rightarrow T = \frac{\lambda}{v_x} = \frac{3.2 \text{ m}}{240 \text{ m/s}} = \underline{.0133 \text{ s}}$$

$$f = \frac{1}{T} = \underline{75 \text{ s}^{-1}}$$

13.

$$(a) \text{ Violet: } f = \frac{1}{T} = \frac{v_x}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = \underline{7.5 \times 10^{14} \text{ s}^{-1}}$$

$$\text{Red: } f = \frac{3 \times 10^8 \text{ m/s}}{700 \times 10^{-9} \text{ m}} = \underline{4.3 \times 10^{14} \text{ s}^{-1}}$$

$$(b) \lambda = \frac{v_x}{f} = \frac{3 \times 10^8 \text{ m/s}}{1.5 \times 10^6 \text{ s}^{-1}} = \underline{200 \text{ m}} \quad \text{for } 1.5 \text{ MHz.}$$

$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{300 \times 10^6 \text{ s}^{-1}} = \underline{1 \text{ m}} \quad \text{for } 300 \text{ MHz.}$$

(Hz = s⁻¹)

$$(c) f = \frac{v_x}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{5 \times 10^{-9} \text{ m}} = \underline{6 \times 10^{16} \text{ s}^{-1}} \quad \text{for } 5 \text{ nm}$$

$$= \frac{3 \times 10^8 \text{ m/s}}{10^{-11} \text{ m}} = \underline{3 \times 10^{19} \text{ s}^{-1}} \quad \text{for } .01 \text{ nm.}$$

14.

$$\omega = 2\pi f = 2\pi \cdot 550 \text{ s}^{-1} = \underline{3460 \text{ s}^{-1}}$$

$$v_x = \frac{\omega}{k} \text{ so } k = \frac{\omega}{v_x} = \frac{3460 \text{ s}^{-1}}{330 \text{ m/s}} = \underline{10.5 \text{ m}^{-1}}$$

So the equation is $y(x,t) = (0.01 \text{ m}) \sin \left[(10.5 \text{ m}^{-1})x + (3460 \text{ s}^{-1})t \right]$

+ because the wave is moving to the left. 