

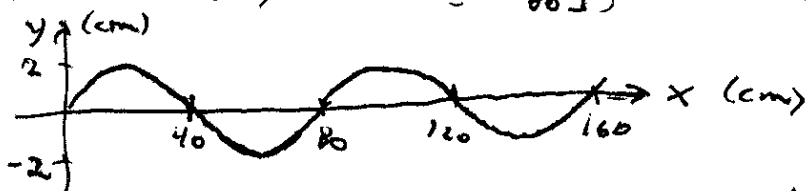
Solutions to Problem Set 11

1. $\omega = 2\pi f = 2\pi \cdot 550 \text{ s}^{-1} = 3460 \text{ s}^{-1}$.

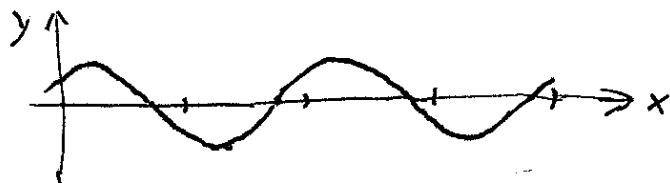
$|v_x| = \frac{\omega}{k}$ so $k = \frac{\omega}{|v_x|} = \frac{3460 \text{ s}^{-1}}{330 \text{ m/s}} = 10.5 \text{ m}^{-1}$.

So the equation is $y(x,t) = (.01 \text{ m}) \sin[(10.5 \text{ m}^{-1})x + (3460 \text{ s}^{-1})t]$
 + because the wave is moving to the left.

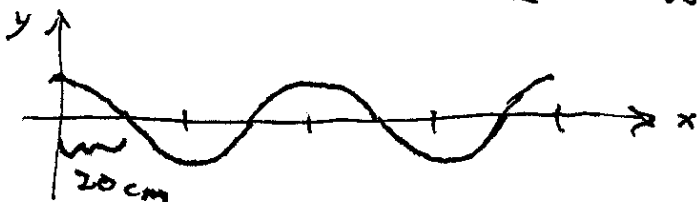
2. (a) At $t=0$, $y = 2 \sin[2\pi \frac{x}{80}]$, so a full cycle is when $x=80$:



(b) At $t=.05 \text{ s}$, $y = 2 \sin[2\pi(\frac{t}{8} + \frac{x}{80})]$. Now at $x=0$, the sine function has already gone through $2 \frac{1}{8}$ cycle.



At $t=.1 \text{ s}$, $y = 2 \sin[2\pi(\frac{t}{4} + \frac{x}{80})]$:



(c) From the graphs, this wave is traveling in the $-x$ direction, with speed $\frac{20 \text{ cm}}{.1 \text{ s}} = 200 \text{ cm/s}$.

(d) Here $k = \frac{2\pi}{80} \text{ cm}^{-1} = .0785 \text{ cm}^{-1}$, $\omega = \frac{2\pi}{.4} \text{ s}^{-1} = 15.7 \text{ s}^{-1}$

and $v_x = -\frac{\omega}{k} = -\frac{15.7 \text{ s}^{-1}}{.0785 \text{ cm}^{-1}} = -200 \text{ cm/s}$. ✓

3. (a) $y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})x - (600 \text{ s}^{-1})t]$.

amplitude = 2.0 mm.

$k = 20 \text{ m}^{-1}$, so $\lambda = \frac{2\pi}{k} = \frac{2\pi}{20} \text{ m} = \underline{.314 \text{ m}}$.

$\omega = 600 \text{ s}^{-1}$, so $f = \frac{\omega}{2\pi} = \underline{95.5 \text{ s}^{-1}}$, $T = \frac{1}{f} = \underline{.0105 \text{ s}}$.

$v_x = + \frac{\lambda}{T} = \underline{30 \text{ m/s}}$, + because of the - inside the sine function.

(b) Transverse speed = $\frac{dy}{dt}$ for any fixed x

$= \frac{d}{dt} [(2 \text{ mm}) \sin[-(600 \text{ s}^{-1})t]] = (2.0 \text{ mm}) \cdot (-1) \cos[-(600 \text{ s}^{-1})t] \cdot (-600 \text{ s}^{-1})$

$= \underline{600} (2 \text{ mm})(600 \text{ s}^{-1}) \cdot \underbrace{\cos[(600 \text{ s}^{-1})t]}_{\text{max} = 1}$

$\rightarrow \text{max. speed} = (2 \text{ mm})(600 \text{ s}^{-1}) = 1200 \text{ mm/s} = \underline{1.2 \text{ m/s}}$.

4.

Let $v_1 = 170 \text{ m/s}$, $v_2 = 180 \text{ m/s}$,

$T_1 = 120 \text{ N}$, $T_2 = ?$

$v_1 = \sqrt{\frac{T_1}{\mu}}$, $v_2 = \sqrt{\frac{T_2}{\mu}}$,

so $v_1^2 = \frac{T_1}{\mu}$, $v_2^2 = \frac{T_2}{\mu} \rightarrow \mu = \frac{T_1}{v_1^2} = \frac{T_2}{v_2^2} \rightarrow T_2 = T_1 \left(\frac{v_2}{v_1}\right)^2$

$\rightarrow T_2 = (120 \text{ N}) \left(\frac{180}{170}\right)^2 = \underline{135 \text{ N}}$.

5. (a) speed = $\sqrt{\frac{\text{tension}}{\text{mass/length}}}$, independent of frequency.

So the speed is unchanged.

$$(b) v = \frac{\lambda}{T} = \lambda f, \quad \text{so } \lambda = \frac{v}{f}.$$

Thus if f is doubled, λ is cut in half.

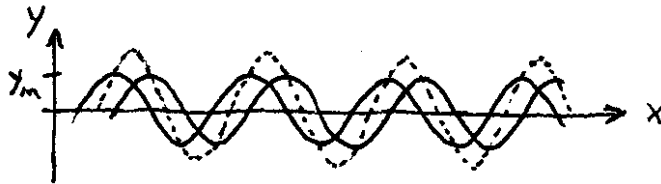
$$6. v_{\text{old}} = \sqrt{\frac{|\vec{F}_{T,\text{old}}|}{m/L}}, \quad v_{\text{new}} = \sqrt{\frac{|\vec{F}_{T,\text{new}}|}{m/L}},$$

$$\text{So } \frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{|\vec{F}_{T,\text{new}}|/(m/L)}}{\sqrt{|\vec{F}_{T,\text{old}}|/(m/L)}} = \sqrt{\frac{|\vec{F}_{T,\text{new}}|}{|\vec{F}_{T,\text{old}}|}} = \sqrt{2} = 1.41$$

↑
since tension doubles

7.


I traced the solid curves from the illustration on page 402, offsetting one by $\frac{1}{4}$ cycle:



The dashed curve is the sum of the two solid curves. Its amplitude appears to be about $1.5 \times$ the amplitude of either solid curve: amplitude $\approx 1.5 \times y_m$.

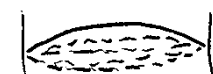
$$\text{Alternatively, from Eq. (17-42), } y'_m = 2y_m \cos\left(\frac{1}{2} \cdot \frac{\pi}{2}\right) = 2y_m \cos \frac{\pi}{4} \\ = \frac{2}{\sqrt{2}} y_m = 1.41 \times y_m.$$


8. (a) speed = $\sqrt{\frac{T}{\mu}} = \sqrt{\frac{150 \text{ N}}{.0072 \text{ kg/m}}} = \underline{144 \text{ m/s}}$.

(b)  $\lambda = \frac{2}{3} L = \frac{2}{3} \times 90 \text{ cm} = \underline{60 \text{ cm}}$.

(c) $f = \frac{v}{\lambda} = \frac{144 \text{ m/s}}{.6 \text{ m}} = 241 \text{ s}^{-1} = \underline{241 \text{ Hz}}$.

9. (a) speed = $\sqrt{\frac{T}{\mu}} = \sqrt{\frac{120 \text{ N}}{.0087 \text{ kg}/1.5 \text{ m}}} = \underline{144 \text{ m/s}}$.

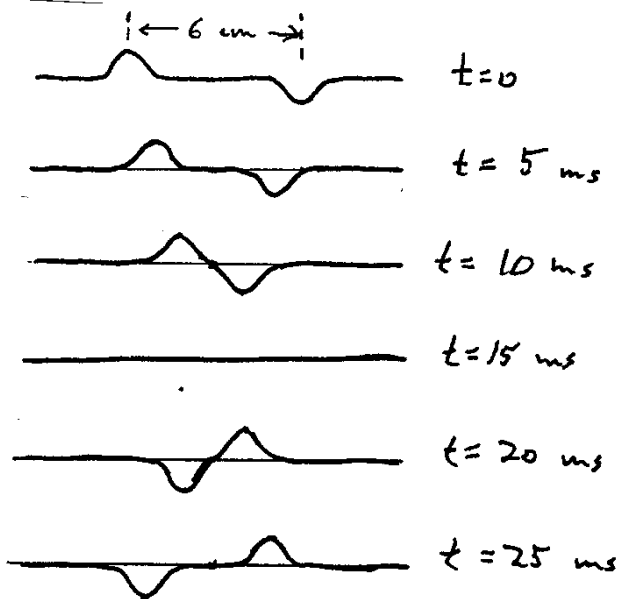
(b)  $\lambda = 2L = \underline{3 \text{ m}}$ (one loop)

 $\lambda = L = \underline{1.5 \text{ m}}$ (two loops)

(c) One loop: $f = \frac{v}{\lambda} = \frac{144 \text{ m/s}}{3 \text{ m}} = \underline{48 \text{ Hz}}$.

Two loops: $f = \frac{144 \text{ m/s}}{1.5 \text{ m}} = \underline{96 \text{ Hz}}$.

10.



In 5 ms, each pulse moves $(2 \text{ m/s})(5 \times 10^{-3} \text{ s}) = .01 \text{ m} = \underline{1 \text{ cm}}$, or $\frac{1}{6}$ the initial distance between them.

At $t = 15 \text{ ms}$, each pulse has reached the mid point so they cancel each other out. Because the string is unstretched, there is no potential energy. The energy must all be kinetic, so the middle part of the string must be moving rapidly.

11.

B has units of $\frac{\text{pressure}}{\text{Volume/Volume}} = \text{pressure} = \frac{N}{m^2}$

So $\sqrt{\frac{B}{\rho}}$ has units of $\sqrt{\frac{N/m^2}{kg/m^3}} = \sqrt{\frac{N \cdot m}{kg}} = \sqrt{\frac{kg \cdot \frac{m^2}{s^2}}{kg}} = \frac{m}{s}$.



12.

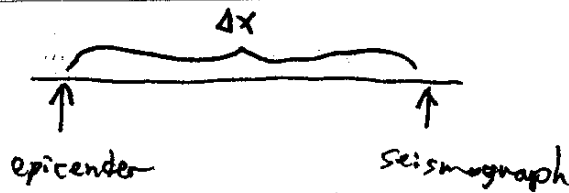
At 20°C, the speed of sound is 343 $\frac{m}{s}$.

Convert to miles: $(343 \frac{m}{s}) (\frac{3.28 ft}{m}) (\frac{1 \text{ mile}}{5280 ft}) = .213 \text{ miles/sec.}$

So in 5 seconds, sound travels 1.07 miles, 7% farther than indicated by this "rule" (The light from the flash travels almost instantaneously.) The time to go one km would be

$\frac{1000 \text{ m}}{343 \text{ m/s}} = 2.92 \text{ s}$, so the lightning is about 1 km away for each three seconds of delay.

13.



For the P waves, $\Delta x = v_p \cdot \Delta t_p$

for the S waves, $\Delta x = v_s \cdot \Delta t_s$

$v_p \Delta t_p = v_s \Delta t_s$

$\rightarrow \frac{\Delta t_s}{\Delta t_p} = \frac{v_p}{v_s} = \frac{8 \text{ km/s}}{4.5 \text{ km/s}} = 1.78$

But we're told that $\Delta t_s = \Delta t_p + 3 \text{ min.}$

$\rightarrow (1.78) \Delta t_p = \Delta t_p + 3 \text{ min.} \rightarrow (1.78) \Delta t_p = 3 \text{ min.} \rightarrow \Delta t_p = 3.86 \text{ min.}$

$\rightarrow \Delta x = (8 \text{ km/s})(231 \text{ s}) = \underline{1850 \text{ km.}}$

$$14. v = \frac{\lambda}{T} = \lambda f \rightarrow \lambda = \frac{v}{f}$$

Lowest pitch: $\lambda = \frac{343 \text{ m/s}}{20 \text{ s}^{-1}} = \underline{17 \text{ m}}$

Highest pitch: $\lambda = \frac{343 \text{ m/s}}{20,000 \text{ s}^{-1}} = \underline{.017 \text{ m} = 1.7 \text{ cm}}$

15.

(a) For a pipe with both ends open, a graph of displacement looks like this:



So $\lambda = 2L$. But $v = \lambda f$

$$\rightarrow v = 2Lf \rightarrow L = \frac{v}{2f} = \frac{343 \text{ m/s}}{2 \cdot 300 \text{ s}^{-1}} = \underline{.57 \text{ m}}$$

(b) The second harmonic of this pipe looks like:



It has half the wavelength, and therefore twice the frequency, or 600 Hz.

For pipe B, the third harmonic looks like:



$$\text{So } L = \frac{5}{4} \lambda = \frac{5}{4} \frac{v}{f} = \frac{5}{4} \frac{343 \text{ m/s}}{600 \text{ s}^{-1}}$$

$$= \underline{.715 \text{ m} = 71.5 \text{ cm}}$$