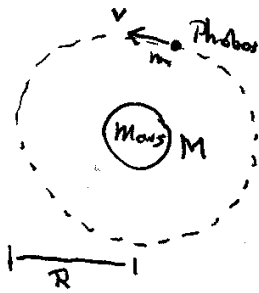


1. $|\vec{F}_g| = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(75 kg)(50 kg)}{(2 m)^2}$

$= 6.25 \times 10^{-8} N \approx \underline{6 \times 10^{-8} N}$. Tiny!

(maybe I'm using the wrong toothpaste...)

2.



$\Sigma \vec{F} = m \vec{a}$

$\rightarrow \vec{F}_g = m \vec{a} \rightarrow |\vec{F}_g| = m |\vec{a}|$

$\rightarrow \frac{GMm}{R^2} = m \frac{v^2}{R}$

$\rightarrow \frac{GM}{R} = v^2 = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^2}{T^2}$

$T = 7h39m$
 $= 459 \text{ min}$
 $= 27,540 s$

$\rightarrow M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 m)^3}{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(27,540 s)^2}$

$= \underline{6.48 \times 10^{23} kg} = 11\% \text{ of Earth's mass}$

3.

Again, $\Sigma \vec{F} = m \vec{a} \rightarrow \frac{GMm}{R^2} = \frac{m v^2}{R}$

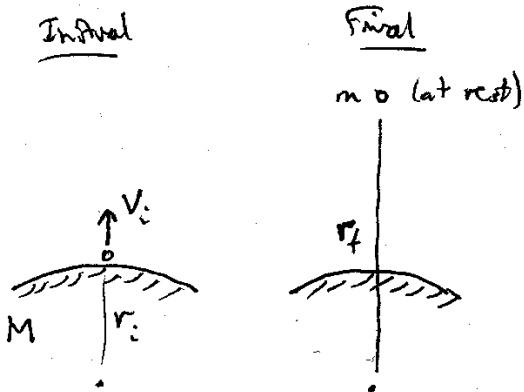
$\rightarrow \frac{GM}{R} = \frac{4\pi^2 R^2}{T^2} \rightarrow \frac{GMT^2}{4\pi^2} = R^3$ (Note $T = 1 \text{ day} = 86,400 s$)

$\rightarrow R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(5.98 \times 10^{24} kg)(86,400 s)^2}{4\pi^2}\right)^{1/3}$

$= 42,250,000 m = 42,250 km$ from center

$= \underline{35,900 km}$ above surface (or 22,300 miles).

4.



$$E_i = E_f$$

$$\rightarrow \frac{1}{2} m v_i^2 - \frac{GMm}{r_i} = 0 - \frac{GMm}{r_f}$$

$$\rightarrow \frac{v_i^2}{2GM} - \frac{1}{r_i} = -\frac{1}{r_f}$$

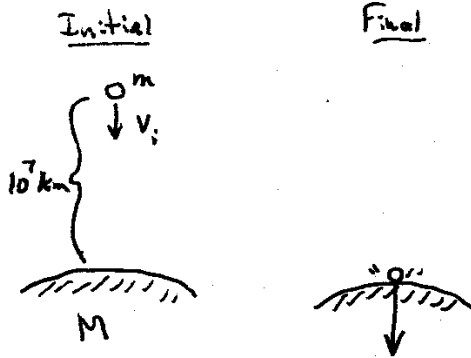
$$\rightarrow \frac{1}{r_f} = \frac{1}{r_i} - \frac{v_i^2}{2GM}$$

$$\rightarrow r_f = \left(\frac{1}{r_i} - \frac{v_i^2}{2GM} \right)^{-1} = \left(\frac{1}{6.37 \times 10^6 \text{ m}} - \frac{(10^4 \text{ m/s})^2}{2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})} \right)^{-1}$$

$$= \left(\frac{1}{6.37 \times 10^6 \text{ m}} - \frac{1}{7.98 \times 10^6 \text{ m}} \right)^{-1} = 3.16 \times 10^7 \text{ m} \quad \text{from center}$$

or $2.52 \times 10^7 \text{ m}$ from surface, i.e., 25,250 km.

5.



$$E_f = E_i$$

$$\rightarrow K_f - \frac{GMm}{r_f} = \frac{1}{2} m v_i^2 - \frac{GMm}{r_i}$$

$$\rightarrow K_f = \frac{1}{2} m v_i^2 + GMm \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$\rightarrow K_f = \frac{1}{2} (3 \times 10^{15} \text{ kg}) (8500 \text{ m/s})^2 + (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) (5.98 \times 10^{24} \text{ kg}) (3 \times 10^{15} \text{ kg})$$

$$\times \left(\frac{1}{6.37 \times 10^6 \text{ m}} - \frac{1}{10^{10} \text{ m}} \right)$$

↑
negligible

$$= 1.08 \times 10^{23} \text{ J} + 1.88 \times 10^{23} \text{ J}$$

$$= 3.0 \times 10^{23} \text{ J} \approx 7 \times 10^7 \text{ 1-megaton bombs.}$$

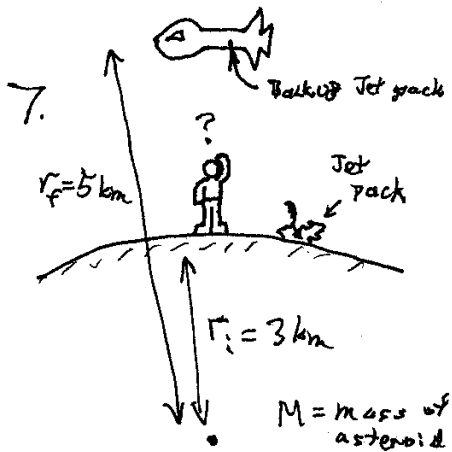
This energy could boil a lot of water, vaporize rocks, eject a lot of stuff into the stratosphere, create seismic waves, etc.

6. "Escape speed" is the speed at which a launched projectile will escape to $r_f = \infty$. So we can repeat #4, setting $r_f = \infty$ and solving for v_i :

$$\frac{v_i^2}{2GM} - \frac{1}{r_i} = \frac{1}{r_f} = \frac{1}{\infty} = 0 \quad (M = \text{mass of moon})$$

$$\rightarrow \frac{v_i^2}{2GM} = \frac{1}{r_i} \rightarrow v_i = \sqrt{\frac{2GM}{r_i}} = \sqrt{\frac{2(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(7.36 \times 10^{22} \text{kg})}{1.74 \times 10^6 \text{m}}}$$

$$= 2375 \text{ m/s} \approx \underline{2.4 \text{ km/s}}$$



In order to jump to the spaceship, I must gain some gravitational energy:

$$\Delta U = U_f - U_i = \left(-\frac{GMm}{r_f}\right) - \left(-\frac{GMm}{r_i}\right)$$

$$= GMm \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

But on earth, the most energy I can put into my jump is

$\Delta U = mgy$, where $y = .5 \text{ m}$. Assuming an equally energetic jump, then, $mgy = GMm \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$. I'll plug in the values of r_i , r_f , and solve for y . Note that

$$M = (\text{density}) \times (\text{volume}) = \rho \cdot \frac{4}{3} \pi r^3 = (4.54 \frac{\text{g}}{\text{cm}^3}) \left(\frac{100 \text{cm}}{1 \text{m}}\right)^3 \left(\frac{1 \text{kg}}{1000 \text{g}}\right) \cdot \frac{4\pi}{3} (3000 \text{m})^3$$

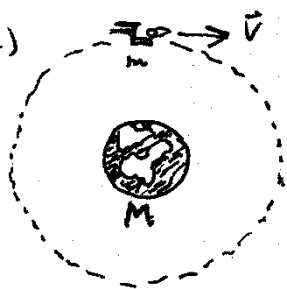
$$= 5.2 \times 10^{24} \text{ kg. So...}$$

$$y_{\text{min}} = \frac{GM}{g} \left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(5.2 \times 10^{24} \text{kg})}{9.8 \text{ N/kg}} \left(\frac{1}{3000 \text{m}} - \frac{1}{5000 \text{m}}\right)$$

$$= (3.49 \times 10^4 \text{m}) \left(\frac{1}{7500 \text{m}}\right) = \underline{0.47 \text{ m}}$$

This is how high I need to be able to jump on earth, so Yes! I can make it!

8. (a)



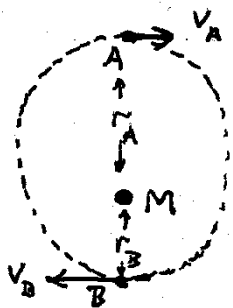
$$\sum \vec{F} = m\vec{a}$$

$$\rightarrow \frac{GMm}{r^2} = m \frac{v^2}{r}$$

$$\rightarrow |v| = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(3 \times 10^{24} \text{ kg})}{10^7 \text{ m}}}$$

$$= 4.47 \times 10^3 \frac{\text{m}}{\text{s}} \approx \underline{4.5 \text{ km/s}}$$

(b)



Energy and angular momentum are both conserved. So apply these laws to points A and B:

$$L_A = L_B$$

$$\rightarrow m v_A r_A = m v_B r_B \rightarrow \frac{v_A}{v_B} = \frac{r_B}{r_A} = \frac{1}{2}$$

$$E_A = E_B \rightarrow \frac{1}{2} m v_A^2 - \frac{GMm}{r_A} = \frac{1}{2} m v_B^2 - \frac{GMm}{r_B}$$

$$\rightarrow \frac{1}{2} v_A^2 - \frac{1}{2} v_B^2 = GM \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

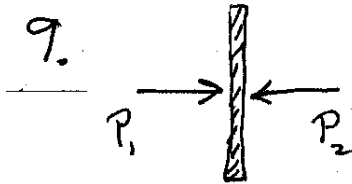
$$\rightarrow \frac{1}{2} v_A^2 - \frac{1}{2} (2v_A)^2 = GM \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$\rightarrow -\frac{3}{2} v_A^2 = GM \left(\frac{1}{r_A} - \frac{2}{r_A} \right)$$

$$\rightarrow v_A^2 = -\frac{2}{3} GM \frac{1}{r_A} (1-2) = +\frac{2}{3} \frac{GM}{r_A}$$

$$\rightarrow v_A = \sqrt{\frac{2}{3} \frac{GM}{r_A}} = \sqrt{\frac{2}{3} \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(3 \times 10^{24} \text{ kg})}{10^7 \text{ m}}}$$

$$= \sqrt{\frac{2}{3}} (4.47 \times 10^3 \frac{\text{m}}{\text{s}}) = 3.65 \times 10^3 \text{ m/s} = \underline{3.7 \text{ km/s}}$$



The net force is

$$\begin{aligned}
 P_1 \cdot A - P_2 \cdot A &= (P_1 - P_2) \cdot A \\
 &= (1 \text{ bar} - .96 \text{ bar}) \cdot (6 \text{ m}^2) \\
 &= (.04 \text{ bar}) (6 \text{ m}^2) \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}} \right) = \underline{24,000 \text{ N}}
 \end{aligned}$$

10. Pressure = $\frac{\text{force}}{\text{area}}$.

Here the force is equal to my weight, $\approx 750 \text{ N}$.

(a) The area of one shoe is about $8 \text{ cm} \times 30 \text{ cm}$ or 240 cm^2 , so for 2 shoes, area = 500 cm^2 .

$$\rightarrow P = \frac{750 \text{ N}}{500 \text{ cm}^2} \cdot \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = \underline{15,000 \text{ N/m}^2}$$

(b) The bottom of a skate blade might measure 2 mm by 25 cm or 5 cm^2 .

$$\rightarrow P = \frac{750 \text{ N}}{5 \text{ cm}^2} \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = 750,000 \text{ N/m}^2$$

(c) My snowshoes measure about $9''$ by $30''$ so the area is $2 \times (9 \text{ in}) \times (30 \text{ in}) \left(\frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^2$

$$\begin{aligned}
 &= .35 \text{ m}^2 \\
 \rightarrow P &= \frac{750 \text{ N}}{.35 \text{ m}^2} \approx \underline{2000 \text{ N/m}^2}
 \end{aligned}$$

