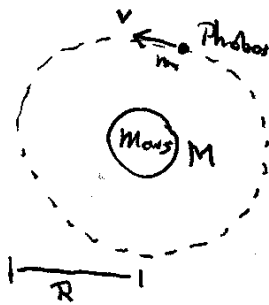


1.  $|\vec{F}_g| = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(75 kg)(50 kg)}{(2 m)^2}$

$= 6.25 \times 10^{-8} N \approx \underline{6 \times 10^{-8} N}$ . Tiny!

(maybe I'm using the wrong toothpaste...)

2.



$\Sigma \vec{F} = m \vec{a}$

$\rightarrow \vec{F}_g = m \vec{a} \rightarrow |\vec{F}_g| = m |\vec{a}|$

$\rightarrow \frac{GMm}{R^2} = m \frac{|\vec{v}|^2}{R}$

$\rightarrow \frac{GM}{R} = |\vec{v}|^2 = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^2}{T^2}$

$\rightarrow M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 m)^3}{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(27,540 s)^2}$

$T = 7h 39min$   
 $= 459 min$   
 $= 27,540 s$

$= \underline{6.48 \times 10^{23} kg} = 11\% \text{ of Earth's mass}$

3.

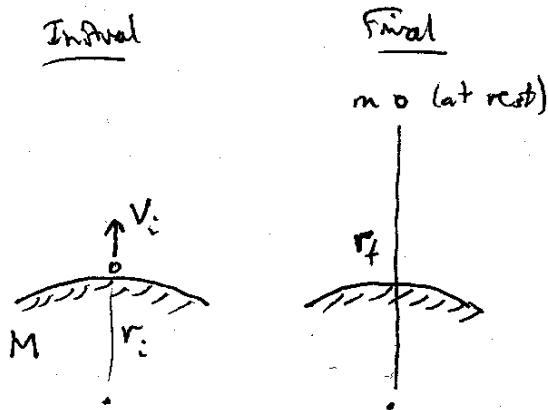
Again,  $\Sigma \vec{F} = m \vec{a} \rightarrow \frac{GMm}{R^2} = \frac{m |\vec{v}|^2}{R}$

$\rightarrow \frac{GM}{R} = \frac{4\pi^2 R^2}{T^2} \rightarrow \frac{GMT^2}{4\pi^2} = R^3$  (Here  $T = 1 \text{ day} = 86,400 s$ )

$\rightarrow R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left(\frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(5.98 \times 10^{24} kg)(86,400 s)^2}{4\pi^2}\right)^{1/3}$

$= 42,250,000 m = 42,250 km$  from center  
 $= \underline{35,900 km}$  above surface (or 22,300 miles).

4.



$$E_i = E_f$$

$$\rightarrow \frac{1}{2} m v_i^2 - \frac{GMm}{r_i} = 0 - \frac{GMm}{r_f}$$

$$\rightarrow \frac{v_i^2}{2GM} - \frac{1}{r_i} = -\frac{1}{r_f}$$

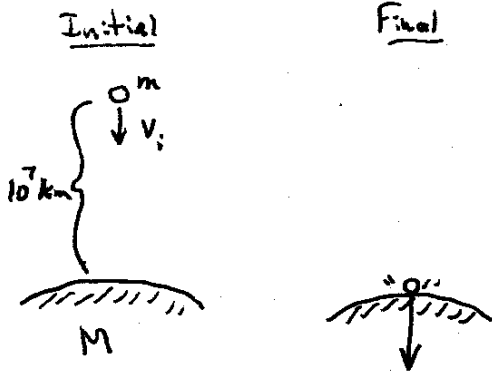
$$\rightarrow \frac{1}{r_f} = \frac{1}{r_i} - \frac{v_i^2}{2GM}$$

$$\rightarrow r_f = \left( \frac{1}{r_i} - \frac{v_i^2}{2GM} \right)^{-1} = \left( \frac{1}{6.37 \times 10^6 \text{ m}} - \frac{(10^4 \text{ m/s})^2}{2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})} \right)^{-1}$$

$$= \left( \frac{1}{6.37 \times 10^6 \text{ m}} - \frac{1}{7.98 \times 10^6 \text{ m}} \right)^{-1} = 3.16 \times 10^7 \text{ m} \quad \text{from center}$$

or  $2.52 \times 10^7 \text{ m}$  from surface, i.e., 25,250 km.

5.



$$E_f = E_i$$

$$\rightarrow K_f - \frac{GMm}{r_f} = \frac{1}{2} m v_i^2 - \frac{GMm}{r_i}$$

$$\rightarrow K_f = \frac{1}{2} m v_i^2 + GMm \left( \frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$\rightarrow K_f = \frac{1}{2} (3 \times 10^{15} \text{ kg}) (8500 \text{ m/s})^2 + (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) (5.98 \times 10^{24} \text{ kg}) (3 \times 10^{15} \text{ kg})$$

$$\times \left( \frac{1}{6.37 \times 10^6 \text{ m}} - \frac{1}{10^{10} \text{ m}} \right)$$

↑  
negligible

$$= 1.08 \times 10^{23} \text{ J} + 1.88 \times 10^{23} \text{ J}$$

$$= 3.0 \times 10^{23} \text{ J} \approx 7 \times 10^7 \text{ 1-megaton bombs.}$$

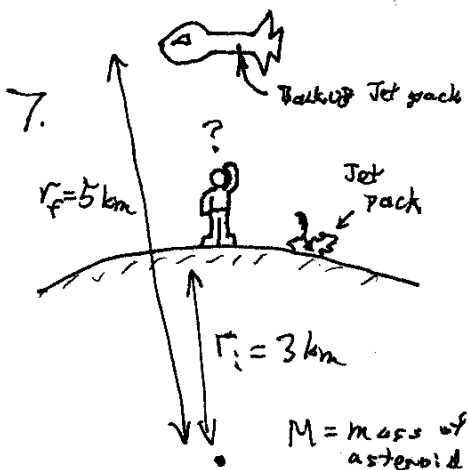
This energy could boil a lot of water, vaporize rock, eject a lot of stuff into the stratosphere, create seismic waves, etc.

6. "Escape speed" is the speed at which a launched projectile will escape to  $r_f = \infty$ . So we can repeat #4, setting  $r_f = \infty$  and solving for  $v_i$ :

$$\frac{v_i^2}{2GM} - \frac{1}{r_i} = \frac{1}{r_f} = \frac{1}{\infty} = 0 \quad (M = \text{mass of moon})$$

$$\rightarrow \frac{v_i^2}{2GM} = \frac{1}{r_i} \rightarrow v_i = \sqrt{\frac{2GM}{r_i}} = \sqrt{\frac{2(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(7.36 \times 10^{22} \text{kg})}{1.74 \times 10^6 \text{m}}}$$

$$= 2375 \text{ m/s} \approx \underline{2.4 \text{ km/s}}$$



In order to jump to the spaceship, I must gain some gravitational energy:

$$\Delta U = U_f - U_i = \left(-\frac{GMm}{r_f}\right) - \left(-\frac{GMm}{r_i}\right)$$

$$= GMm \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

But on earth, the most energy I can put into my jump is  $\Delta U = mgy$ , where  $y = .5 \text{ m}$ . Assuming an equally energetic jump, then,

$mgy = GMm \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$ . I'll plug in the values of  $r_i$ ,  $r_f$ , and solve for  $y$ . Note that

$$M = (\text{density}) \times (\text{volume}) = \rho \cdot \frac{4}{3} \pi r^3 = (4.54 \frac{\text{g}}{\text{cm}^3}) \left(\frac{100 \text{cm}}{1 \text{m}}\right)^3 \left(\frac{1 \text{kg}}{1000 \text{g}}\right) \cdot \frac{4\pi}{3} (3000 \text{m})^3$$

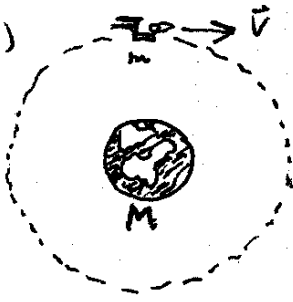
$$= 5.2 \times 10^{24} \text{ kg. So ...}$$

$$y_{\text{min}} = \frac{GM}{g} \left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(5.2 \times 10^{24} \text{kg})}{9.8 \text{ N/kg}} \left(\frac{1}{3000 \text{m}} - \frac{1}{5000 \text{m}}\right)$$

$$= (3490 \text{m}^2) \left(\frac{1}{7500 \text{m}}\right) = \underline{0.47 \text{ m}}$$

This is how high I need to be able to jump on earth so YEs! I can make it!

8. (a)



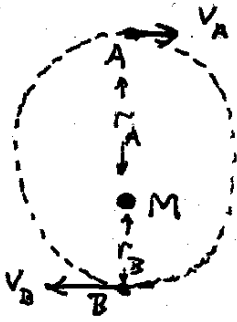
$$\sum \vec{F} = m\vec{a}$$

$$\rightarrow \frac{GMm}{r^2} = m \frac{|\vec{v}|^2}{r}$$

$$\rightarrow |\vec{v}| = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(3 \times 10^{24} \text{ kg})}{10^7 \text{ m}}}$$

$$= 4.47 \times 10^3 \frac{\text{m}}{\text{s}} \approx \underline{4.5 \text{ km/s.}}$$

(b)



Energy and angular momentum are both conserved. So apply these laws to points A and B:

$$L_A = L_B$$

$$\rightarrow m v_A r_A = m v_B r_B \rightarrow \frac{v_A}{v_B} = \frac{r_B}{r_A} = \frac{1}{2}$$

$$E_A = E_B \rightarrow \frac{1}{2} m v_A^2 - \frac{GMm}{r_A} = \frac{1}{2} m v_B^2 - \frac{GMm}{r_B}$$

$$\rightarrow \frac{1}{2} v_A^2 - \frac{1}{2} v_B^2 = GM \left( \frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$\rightarrow \frac{1}{2} v_A^2 - \frac{1}{2} (2v_A)^2 = GM \left( \frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$\rightarrow -\frac{3}{2} v_A^2 = GM \left( \frac{1}{r_A} - \frac{2}{r_A} \right)$$

$$\rightarrow v_A^2 = -\frac{2}{3} GM \frac{1}{r_A} (1-2) = +\frac{2}{3} \frac{GM}{r_A}$$

$$\rightarrow v_A = \sqrt{\frac{2}{3} \frac{GM}{r_A}} = \sqrt{\frac{2}{3} \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(3 \times 10^{24} \text{ kg})}{10^7 \text{ m}}}$$

$$= \sqrt{\frac{2}{3}} (4.47 \times 10^3 \frac{\text{m}}{\text{s}}) = 3.65 \times 10^3 \text{ m/s} = \underline{3.7 \text{ km/s.}}$$

9. (a) For any mass on a spring,  $T = 2\pi\sqrt{\frac{m}{k}}$ .

But here  $m$  should be the total,  $M+m$ ,

$$\text{So } T = 2\pi\sqrt{\frac{M+m}{k}} \rightarrow T^2 = 4\pi^2 \left(\frac{M+m}{k}\right)$$

$$\rightarrow \frac{T^2 k}{4\pi^2} = M+m \rightarrow M = \left(\frac{k}{4\pi^2}\right)T^2 - m \quad \checkmark$$

(b) When  $M=0$ ,  $T = .90149 \text{ s}$ .

$$\text{So } m = \frac{T^2 k}{4\pi^2} = \frac{(.90149 \text{ s})^2 (605.6 \frac{\text{N}}{\text{m}})}{4\pi^2} = 12.47 \text{ kg}.$$

$$(c) M = \frac{T^2 k}{4\pi^2} - m = \frac{(2.08832 \text{ s})^2 (605.6 \frac{\text{N}}{\text{m}})}{4\pi^2} - 12.47 \text{ kg}$$

$$= 66.90 \text{ kg} - 12.47 \text{ kg} = \underline{54.4 \text{ kg}} \quad (\approx 120 \text{ lbs})$$

10.  $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$  So system 1's frequency is smaller by a factor of  $\sqrt{2}$ .

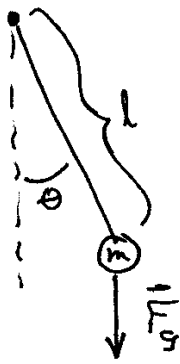
Similarly,  $T \propto \sqrt{m}$ , so ~~System 1's~~ period is greater by  $\sqrt{2}$ .

Total energy =  $\frac{1}{2}kA^2$ , same for both. (They're the same when the springs are fully stretched, therefore they're always the same!)

Max. K.E. = same for both, since total energy is the same.

But  $K = \frac{1}{2}m|v|^2 \rightarrow |v| = \sqrt{\frac{2K}{m}}$ , so S2's max speed is less by  $\sqrt{2}$ .

11.



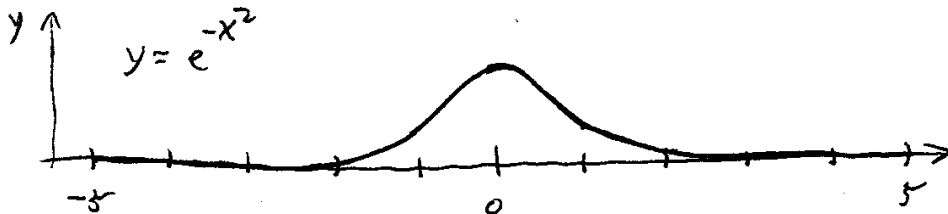
The torque from gravity is

$$\tau = l \cdot |\vec{F}_g| \cdot \sin\theta.$$

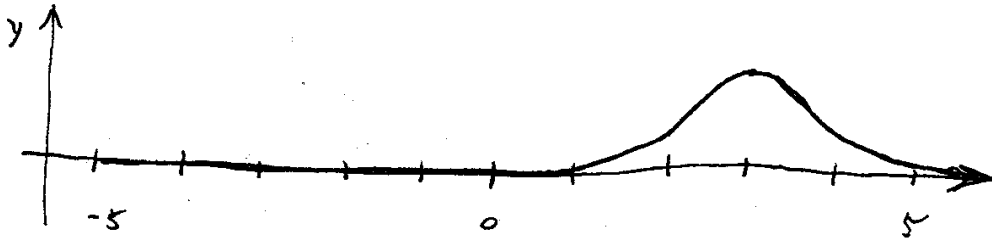
When  $\theta$  is small, doubling  $\theta$  causes  $\sin\theta$  to almost double, so the torque grows in proportion to  $\theta$ , like a spring force ( $-k_s x$ ).

So for small angles, doubling the amplitude approximately doubles the torque, allowing the pendulum to compensate for the greater distance by swinging faster (greater acceleration). When  $\theta$  gets large, however,  $\sin\theta$  no longer grows in direct proportion to  $\theta$ , so the torque doesn't grow fast enough to compensate for the greater distance. In fact,  $\sin\theta$  stops growing at  $\theta = 90^\circ$ .

12. (a)



(b)



(c) The second function is  $y = e^{-(x-3)^2}$ .

Now, to get the same value of  $y$  as before, you have to plug in a value of  $x$  that is larger by 3 units.