

**Problem Set 15**  
(due Thursday, April 24)

1. Explain how you can tell that the “multiplicity” (and hence the entropy) of each of these systems increases during the process described:
  - (a) A gas expands to occupy a larger volume, with no change in temperature;
  - (b) The temperature of a gas increases, with no change in volume;
  - (c) An ice cube melts;
  - (d) Humpty Dumpty has a great fall.
2. A cup (250 g) of water (already at 100°C) boils into steam. By how much does its entropy change? Another cup of water, at 0°C, freezes into ice. By how much does its entropy change?
3. The temperature inside your house is 25°C, while the temperature outside is 0°C. Over the course of an hour, 15 MJ (megajoules) of heat escape from inside to outside. The temperatures of the interior and exterior do not change significantly. (a) By how much does the entropy of the outdoors change due to this process? (b) By how much does the entropy of the indoors change due to this process? (c) What is the net change in the entropy of the universe due to this process? (Check that the net change is positive, as it must be.)
4. An unrealistic, but mathematically simple, heat engine works as follows. A mole of air (5 degrees of freedom per molecule), initially at room temperature and atmospheric pressure, is heated at constant volume until its pressure doubles. Then the air undergoes a constant-pressure expansion until its volume doubles. Third, the pressure is reduced to its original value while holding the volume constant. Finally, the volume is reduced to its original value, holding the pressure constant. (a) Draw a  $PV$  diagram for this cycle. (b) For each of the four steps in the cycle, compute the work done by the gas, the change in the internal energy of the gas, and the heat added to the gas. Express your answers in Joules. (c) Compute the total  $W$ ,  $\Delta E$ , and  $Q$  for the whole cycle. (d) Compute the efficiency of this heat engine. (e) Compute the efficiency of an ideal engine that operates between the same extremes of temperature.
5. Consider an engine that absorbs heat from a reservoir at 400°C, and expels waste heat to a reservoir at 25°C. In one second, the heat absorbed is one million joules. (a) How much entropy enters the engine in one second, along with this heat? (b) If this same amount of entropy is to be expelled to the low-temperature reservoir, how much heat must go along with it? (c) After expelling the waste heat, how much energy is left to do work? (d) Use your answer to part (c) to compute the efficiency of this engine, then check that you get the same result using the formula  $1 - T_c/T_h$ .
6. At a power plant that produces 1 gigawatt of electricity, the steam turbines take in steam at a temperature of 500°C, and the waste heat is expelled into a river at 20°C.

- (a) What is the maximum possible efficiency of this plant? (b) At what minimum rate is waste heat expelled into the river? If the average flow rate of the river is 100,000 liters per second, by how much will its temperature increase? (c) Suppose you invent a new material for making pipes and turbines, which allows the maximum steam temperature to be raised to  $600^{\circ}\text{C}$  instead of  $500$ . Approximately how much money can you make in a year by installing your improved hardware, if you sell the additional electricity for 5 cents per kilowatt-hour? (Assume that the amount of fuel consumed at the plant remains the same as before.)
7. If you leave your refrigerator door open, what will happen to the temperature in your kitchen? Explain carefully.
  8. Suppose that heat leaks into your kitchen refrigerator at an average rate of 200 watts. Assuming ideal operation, how much power must it draw from the wall? (Use any reasonable values for the temperature inside and outside the refrigerator.)

*A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. Yet I was asking something which is about the scientific equivalent of: Have you read a work of Shakespeare's?*

—C. P. Snow, *The Two Cultures*

## Study Guide

You should understand the terms **isothermal** and **adiabatic**, but you need not memorize any of the formulas that apply to ideal gases undergoing isothermal or adiabatic expansion/compression.

The **multiplicity** of a system is the number of possible ways in which all its parts (atoms, electrons, units of energy, etc.) could be arranged. Configurations with greater multiplicity are inherently more likely to occur. For mathematical convenience, we usually work with the natural logarithm of the multiplicity, and for historical reasons, we multiply the logarithm by Boltzmann's constant. The result is called the **entropy** ( $S$ ):

$$S = k_B \ln(\text{multiplicity}).$$

When heat  $Q$  flows into an object at temperature  $T$ , its entropy increases by  $Q/T$ . In fact, entropy was originally defined as the thing that changes by  $Q/T$  whenever an amount of heat equal to  $Q$  flows into the system and the system's temperature is  $T$ :

$$\Delta S = \frac{Q}{T}.$$

When  $Q$  is negative (so heat flows *out* of the system), its entropy decreases. Obviously this equation does not tell us what entropy actually *is*. Nevertheless, you should be able to use this formula to compute entropy changes caused by heat flow. (Entropy can also change for other reasons; then this formula cannot be used directly to compute  $\Delta S$ .)

The **second law of thermodynamics** states that entropy (or multiplicity) tends to increase. That is, processes that cause the total entropy of the universe to increase will happen spontaneously, while processes that would decrease the total entropy of the universe cannot occur. Fundamentally, this law is true for statistical reasons: high-entropy configurations of particles and energy are overwhelmingly more probable than low-entropy configurations.

A **heat engine** is a device that absorbs heat energy (or the equivalent) and converts part of this energy to work. You should know that the efficiency of such an engine is defined as  $W_{\text{net}}/Q_{\text{input}}$ , and understand why it is defined in this way. You should (in terms of entropy and the second law) understand why all engines must produce some waste heat. Because of this waste heat, the highest possible efficiency for an engine operating between a high temperature of  $T_H$  and a low temperature of  $T_L$  is  $1 - (T_L/T_H)$ .

A **refrigerator** is a device that moves heat “backwards”, from a cooler place to a hotter place. In order to make this happen, some work input is required; you should understand why (in terms of entropy and the second law). You should know that the “coefficient of performance” of a refrigerator is defined as  $Q_{\text{input}}/W_{\text{net}}$ , and understand why it is defined in this way. The highest possible coefficient of performance for a refrigerator operating between temperatures  $T_H$  and  $T_L$  is  $T_L/(T_H - T_L)$ .