

Problem Set 13

(due Friday, April 8)

1. Estimate how long it should take to bring a cup of water to boiling temperature in a typical 600-watt microwave oven. (Use any reasonable values for the amount of water and its initial temperature.) Explain why there is technically no heat involved in this process.
2. A certain substance has a molar mass of 50 g. When 314 J of heat is added to a 30 g sample of this material, its temperature rises from 25°C to 45°C. (a) What is the specific heat of this substance? (b) How many moles of the substance are in the sample? (c) What is the molar specific heat of the substance?
3. A 75-kg physics professor wishes to hike from the Weber State campus (elevation 4700 feet) to the summit of Mt. Ogden (elevation 9572 feet). (a) Assuming that he is 25% efficient at converting chemical energy (from food) into mechanical work, how many bowls of corn flakes (standard serving size 1 oz., 100 kilocalories) should he eat first? (b) As the physics professor climbs the mountain, three-quarters of the energy from the corn flakes is converted to thermal energy. If there were no way to dissipate this energy, by how many degrees would the professor's body temperature increase? (c) In fact, the extra energy does not warm the professor's body significantly; instead, it goes (mostly) into evaporating water from his skin. How many liters of water should he drink during the hike to replace the lost fluids? (At 25 °C, a reasonable temperature to assume, the heat of vaporization of water is 2.44×10^6 J/kg, slightly more than at 100 °C.)
4. A beaker of water is initially at 30°C. The beaker is made of 100 g of aluminum (whose specific heat you can look up) and contains 180 g of water. (a) Suppose you add 100 g of ice to the beaker. When the system again comes to equilibrium, what is the final temperature? If the temperature is 0°C, how much ice remains? (Assume that no heat escapes from the system.) (b) Repeat the analysis for the case where you add only 50 g of ice.
5. A gas is compressed at a constant pressure of 0.80 atm, from 9.0 liters down to 2.0 liters. In the process, 400 J of heat leaves the gas. (a) What is the work done on the gas? (b) What is the change in its energy?
6. How much work is done by the steam when one mole of water at 100°C boils to become steam (at the same temperature, and at atmospheric pressure)? What is the change in the energy content of the H₂O during this process? (Hint: to answer the second question you need to know the latent heat of vaporization.)
7. How much work is required to compress 10 liters of air at atmospheric pressure down to a volume of one liter, assuming that the temperature is held constant at 300 K? What happens to the energy content of the air during this process? How much heat must be added or removed during this process?
8. In a previous problem set you estimated the number of air molecules in your living room. What is the average translational kinetic energy of each of these molecules, expressed in joules and in electron-volts? (An electron-volt is the amount of energy that a one-volt battery gives to each electron that it pushes around a circuit, 1.6×10^{-19} J.) What is the total translational kinetic energy of all the molecules (in joules)?

9. Consider an oxygen molecule (O_2) and a hydrogen molecule (H_2), both belonging to a gas at some well-defined temperature. On average, which molecule has more translational kinetic energy? On average, which molecule is moving faster? By what factor?
10. The heat capacity at constant volume of one mole of water vapor (at 300 K) is 27.0 J/K. Use this fact to estimate the number of degrees of freedom per molecule. Describe what each of the degrees of freedom might be.
11. Two moles of oxygen gas are heated from 20°C to 40°C. How much heat is transferred to the gas if the process occurs at (a) constant volume; (b) constant pressure? Explain in your own words why the two answers are different. (You may use theoretical values of the heat capacities.)
12. An ideal gas is in a cylinder, initially at room temperature and occupying a volume V_i . My task is to push on the piston to compress the gas to a smaller volume, V_f . Being lazy, I wish to do as little work as possible. Should I compress the gas quickly, so there is no time for heat to flow out of the gas, or should I compress it slowly, so that the gas always remains essentially at room temperature? Explain carefully.

Study Guide

Heat, in physics, is a spontaneous flow of energy from one object to another, caused by a difference in their temperatures. The symbol for the amount of heat energy that flows into an object is Q . Since all forms of energy transfer are classified as either heat or work, we can write the total change in the energy of a system as

$$\Delta E = Q + W.$$

This equation is usually called the **first law of thermodynamics**. Here Q and W are conventionally defined to be positive when that type of energy enters the system, and negative when energy leaves the system.

The **specific heat** of a substance is defined as the amount of energy needed to raise its temperature, per degree, per unit mass:

$$c = \frac{\text{Energy supplied}}{M \Delta T}.$$

Despite the word “heat” in the name, this definition does not assume that the energy enters as heat—it could also enter as work. This definition is *ambiguous*, though, because the energy that *you* supply may or may not equal the change in the system’s energy, ΔE . The most important exception is when the system expands and performs work on its surroundings. Therefore the specific heat will depend on whether or not the system’s volume is held fixed as you add energy.

The **molar specific heat** of a substance is the same thing, but per mole instead of per unit mass:

$$C = \frac{\text{Energy supplied}}{n \Delta T}.$$

(This notation follows Knight’s book; some texts use capital C for the *total* heat capacity of an object, Mc .)

The specific heat of water is 4.186 J/g·°C. This amount of energy is also called a calorie; the familiar food calorie is really a kilocalorie, or 4186 J.

During a phase change such as melting or boiling, the temperature does not change and so c and C are technically infinite. In this context we define the **latent heat**, L , to be the energy input needed to accomplish the change, per unit mass: $L = (\text{Energy supplied})/M$. In this case it is always assumed that the pressure is held fixed, and that the energy supplied does not include the (usually negative) work done by the surroundings as the system’s volume changes. For melting ice, the latent heat is 80 cal/g; for boiling water it is 540 cal/g.

The **equipartition theorem** relates molecular energy to temperature for many, but not all, forms of energy. It says that each “degree of freedom” has an average energy of $\frac{1}{2}k_B T$. Examples of degrees of freedom include translational motion along any axis, rotational motion about any axis, and vibrational kinetic and potential energy. You should be able to correctly count degrees of freedom for simple gases and solids, realizing that some degrees of freedom might be “frozen out” at any given temperature. The most common application of equipartition is to a molecule’s translational kinetic energy, which in three dimensions has an average value of $\frac{3}{2}k_B T$. From this formula you can easily show that the rms (root-mean-square) speed of a molecule is $v_{\text{rms}} = \sqrt{3k_B T/m}$ (where m is the molecule’s mass).

You should understand the terms **isothermal** and **adiabatic**, but you need not memorize any of the formulas that apply to ideal gases undergoing isothermal or adiabatic expansion/compression.