

Problem Set 12

(due Wednesday, April 5)

1. A traveling wave on a string is described by the formula

$$y = A \sin \left[2\pi \left(\frac{t}{t_0} + \frac{x}{x_0} \right) \right],$$

where $A = 2.0$ cm, $t_0 = 0.40$ s, and $x_0 = 80$ cm. (a) For $t = 0$, plot y as a function of x over the range 0 to 160 cm. (b) Repeat part (a) for $t = 0.05$ s and for $t = 0.10$ s. (c) From your graphs, what is the wave speed, and in which direction is the wave traveling? (d) Find the angular wavenumber and angular frequency of this wave, and show how to calculate the speed from these two quantities.

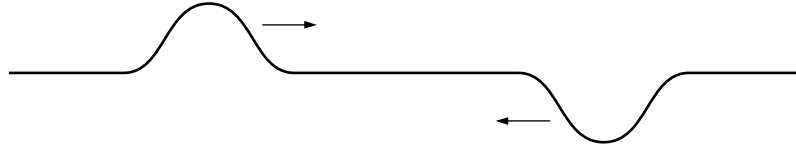
2. The equation of a transverse wave traveling along a string is given by

$$y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})x - (600 \text{ s}^{-1})t].$$

(a) Find the amplitude, frequency, velocity, and wavelength of the wave. (b) Find the maximum transverse speed of a particle in the string.

3. The speed of a transverse wave on a string is 170 m/s when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/s?
4. Suppose you are wiggling the end of a long string, to produce a sinusoidal wave. After a while, you double the frequency of your wiggles, without changing the string tension. (a) What happens to the speed of the waves produced? (b) What happens to the wavelength of the waves produced?
5. A long wire is held fixed at both ends, under tension. If you double the tension without changing the length, what is the ratio of the new wave speed to the old wave speed, for transverse waves traveling along the wire?
6. Two identical sinusoidal traveling waves, moving in the same direction, are out of phase (i.e., offset from each other) by $\frac{\pi}{2}$ radians (1/4 cycle). These waves combine to form a single resultant wave. How does the amplitude of the resultant wave compare to the amplitudes of the original waves? (Please answer this question by *carefully* plotting both waves, then adding the waves graphically and measuring the resulting amplitude with a ruler. You may optionally wish to check your answer using a formula from the text, but this is not required.)
7. A nylon guitar string has a length of 90 cm (between the fixed supports), a tension of 150 N, and a mass per unit length of 7.2 g/m. (a) What is the speed of traveling waves on this string? (b) Suppose that the string is oscillating in a standing wave pattern with two nodes (hence three “bumps”). What is the wavelength of this standing wave? (c) What is the frequency of the standing wave?
8. A 1.5 meter wire has a mass of 8.7 g and is held under a tension of 120 N. The wire is held from both ends and set into vibration. Calculate (a) the speed of waves on the wire; (b) the wavelengths of the two longest-wavelength sinusoidal standing waves; and (c) the frequencies of these standing waves.

9. Two pulses travel along a string in opposite directions, as shown below. (a) If the wave speed is 2.0 m/s and the peaks of the pulses are 6.0 cm apart at $t = 0$, sketch the patterns when $t = 5, 10, 15, 20,$ and 25 milliseconds. (b) What has happened to the energy of the pulses at $t = 15$ ms?



10. Show that the formula $\sqrt{B/\rho}$ for the speed of sound has the correct units.
11. A rule for finding your distance in miles from a lightning flash is to count seconds from the time you see the flash until you hear the thunder, then divide the count by five. (a) Explain this rule and determine the percent error in it at 20°C , assuming that the sound travels to you along a straight path. (b) Devise a similar rule for obtaining the distance in kilometers.
12. Earthquakes generate sound waves inside the earth. Unlike a gas, a solid can transmit both transverse (S) and longitudinal (P) waves. In typical locations, the speed of the S waves is about 4.5 km/s and the speed of the P waves is 8.0 km/s. A seismograph records P and S waves from an earthquake. The first P waves arrive 3.0 minutes before the first S waves. Assuming that the waves traveled in a straight line, how far away did the earthquake occur?
13. The audible frequency range for normal hearing is from about 20 Hz to 20 kHz. What are the wavelengths (in air) of sound waves at those frequencies?
14. Organ pipe A, with both ends open, has a fundamental frequency of 300 Hz. The third harmonic of pipe B, with one end open, has the same frequency as the second harmonic of pipe A. How long are each of the pipes?
15. Two loudspeakers are aimed eastward, separated in the N-S direction by 2.0 meters. They produce identical signals, in phase with each other. A listener is positioned 3.75 m east of one of the speakers (and is therefore somewhat farther away from the other). For what frequencies in the audible range does the listener hear a minimum-intensity signal? For what frequencies is the intensity a maximum?

Study Guide

You should understand the following terms as applied to waves: transverse; longitudinal; amplitude; frequency; angular frequency; period; wavelength; angular wave number; speed; superposition; interference.

Given a mathematical formula for a pulse or periodic wave traveling in one dimension (as on a string), you should be able to determine any of the relevant quantities (amplitude, frequency, etc.) in the preceding list. Given a sufficient number of these quantities, you should be able to write down a mathematical formula describing the pulse or wave.

For transverse waves traveling along a taut string, Newton's laws predict that the speed should be

$$\text{speed} = \sqrt{\frac{\text{tension}}{\text{mass/length}}}.$$

Thus the tighter the string, the faster the wave travels, while the heavier the string (per unit length), the slower the wave travels.

You should understand the principle of superposition as applied to waves and wave pulses. You should be able to determine when two waves will cancel completely ("destructive interference") and when they will add to a maximum resultant wave ("constructive interference").

You should be able to draw standing-wave patterns and determine the various standing-wave wavelengths and frequencies for strings and pipes (with open and/or closed ends).

You should understand what sound waves are (in terms of density and pressure variations), and how their speed depends on the density of the medium and its resistance to being compressed. You should memorize the approximate value of the speed of sound in air at room temperature (340 m/s).