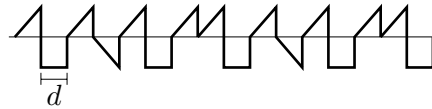
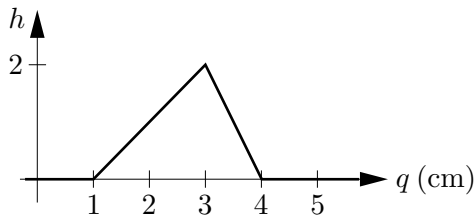


Problem Set 11
(due Wednesday, March 29)

1. Elemental carbon has two possible crystal structures, diamond and graphite. Graphite is more stable at low pressures (implying that diamonds are *not* forever, though they last thousands of years if they are not heated), while diamond is more stable at high pressures. To convert graphite into diamond requires a pressure of approximately 60,000 atmospheres. At what depth below earth's surface would you expect to find such a pressure, assuming that rock can be treated as a fluid (at great depths) and that the average rock has a density three times that of water?
2. If you swim in the Dead Sea, about 1/3 of your body will be above the water line. Assuming that your body has the same density as pure water (a good enough approximation), what is the density of the water in the Dead Sea? (Of course, you can observe the same effect, to a lesser extent, in the Great Salt Lake.)
3. A garden hose with an internal diameter of 3/4 inch is connected to a sprinkler consisting of an enclosure with 24 holes, each 0.050 inch in diameter. If the water in the hose is moving at 3 ft/s, at what speed does it leave the holes in the sprinkler?
4. Suppose you were to drill a hole near the base of Hoover Dam, 700 ft below the water level of Lake Mead. At approximately what speed would the water come rushing out of the hole? (Hint: Use energy conservation.)
5. The windows in an office building each measure 4 m by 5 m. On a stormy day, air is blowing at 30 m/s past a window on the top floor. Estimate the net force on the window due to the air inside and outside, using Bernoulli's principle. (Take the density of the air to be 1.23 kg/m³.)
6. How do astronauts "weigh" themselves in space? They sit in a chair attached to a large spring, and measure the period of oscillation. (a) If M is the mass of the astronaut and m is the effective mass of the chair and the moving end of the spring, show that $M = (k/4\pi^2)T^2 - m$, where T is the period of oscillation and k is the spring constant. (b) The spring constant of the device used on Skylab was 605.6 N/m, and the period of oscillation of the empty chair was 0.90149 s. Calculate the effective mass of the chair. (c) With an astronaut in the chair, the period of oscillation was 2.08832 s. Calculate the mass of the astronaut.
7. Suppose you have two identical springs, with different masses attached, each oscillating with the same amplitude. The mass for System 1 is twice that of System 2. Discuss how the following quantities differ for the two systems: frequency, period, total energy, maximum speed, maximum kinetic energy.
8. The fact that g varies from place to place over earth's surface drew attention when Jean Richer in 1672 took a pendulum clock from Paris to Cayenne, French Guiana,

and found that it lost 2.5 minutes per day. If $g = 9.81 \text{ m/s}^2$ in Paris, what is g in Cayenne?

9. (a) Sketch a graph of the function $y = e^{-x^2}$, for values of x ranging from -5 to 5 . (b) Sketch a graph of a function that is the same as in part (a), but displaced to the right by 3 units. (c) Write down a formula for the function that you plotted in part (b), and explain in English why you modified the original formula in the way that you did.
10. The figure below (left) shows a graph of the function $h(q)$, where q is any number with units of distance, assumed to be in centimeters. Suppose, now, that a wave pulse is described by the function $h(x - bt)$, where the constant b equals 5 cm/s . What are (a) the speed and (b) the direction of travel of this pulse? Justify your answers carefully. (c) Plot this wave as a function of x at $t = 2 \text{ s}$. (d) Plot this wave as a function of t for $x = 10 \text{ cm}$.



11. The figure above (right) shows a strange periodic wave. What is its wavelength, in terms of the parameter d ?
12. A wave has a speed of 240 m/s and a wavelength of 3.2 m . What are the (a) frequency and (b) period of this wave?
13. The speed of electromagnetic waves in vacuum is $3.0 \times 10^8 \text{ m/s}$. (a) Wavelengths of visible light waves range from about 400 nm (violet) to about 700 nm (red). What is the range of frequencies of visible light waves? (b) The range of frequencies for shortwave radio (including FM and VHF television) is 1.5 MHz to 300 MHz . What is the corresponding wavelength range? (c) X-rays are also electromagnetic waves. Their wavelength range extends from about 5.0 nm to about 0.01 nm . What is the corresponding frequency range?
14. Write an equation for a sinusoidal wave traveling in the negative direction along the x axis and having an amplitude of 0.010 m , a frequency of 550 Hz , and a speed of 330 m/s .

Study Guide

You should understand the concepts of density and pressure, including the SI units of these quantities. You should be able to convert between SI units and related units such as g/cm^3 , kg/liter , bars, and atmospheres. You should know the densities of water and air (at sea level), and the value of atmospheric pressure, to one significant figure.

In a static fluid, the pressure exerted on the container or on a submerged object depends only on the depth. If P_0 is the pressure at the top surface, and an object is submerged a distance h below this surface, then

$$P(h) = P_0 + \rho gh,$$

where ρ is the density of the fluid (assumed here to be uniform).

Because the pressure below a submerged object is always greater than the pressure above, there is net force exerted upward by the fluid. This net force is called the *buoyant force*. Since the buoyant force on a chunk of the fluid itself must precisely balance the weight of the chunk, and since the buoyant force on an object depends only on its size and shape, not on what it's made out of, we can conclude that the buoyant force on any submerged object is equal in magnitude to the weight of the fluid displaced (Archimedes' principle).

When a fluid flows smoothly through a pipe or other enclosed container, conservation of the *amount* of fluid implies the *continuity equation*,

$$\rho Av = \text{constant},$$

where A is the cross-sectional area and v is the fluid's speed. Conservation of mechanical energy (which strictly applies only to fluids with no viscosity and no turbulence) implies the *Bernoulli equation*,

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}.$$

Here P is the pressure and y is the height above some arbitrary reference point.

Oscillatory Motion

For a simple mass-spring system, the differential equation of motion (derived from Newton's second law) is

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x.$$

A solution to this equation is the function

$$x(t) = A \cos \omega t,$$

where A (the “amplitude”) can be any distance and ω (the “angular frequency”) is $\sqrt{k/m}$. (A sine function with the same angular frequency is also a solution.) The period of oscillation (time to go back and forth once) is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}},$$

and the ordinary frequency is $f = 1/T = \omega/2\pi$.

Many other systems oscillate in a similar way, and the mathematics is exactly the same. Whenever the equation of motion says that the acceleration is minus some constant times the position, you can conclude that the motion obeys a cosine or sine function with ω equal to the square root of that constant. An important special case is a simple pendulum of length L , whose period is $2\pi\sqrt{L/g}$.

Waves

You should understand the following terms as applied to waves: transverse; longitudinal; amplitude; frequency; angular frequency; period; wavelength; angular wave number; speed; superposition; interference.

Given a mathematical formula for a pulse or periodic wave traveling in one dimension (as on a string), you should be able to determine any of the relevant quantities (amplitude, frequency, etc.) in the preceding list. Given a sufficient number of these quantities, you should be able to write down a mathematical formula describing the pulse or wave.