

**Problem Set 10**  
(due Wednesday, March 9)

1. Estimate the gravitational force between you and a person sitting 2 m away from you. (You should find that the attraction is negligible. As Einstein once said, “Gravity cannot be held responsible for people falling in love.”)
2. The planet Mars has a satellite (moon), called Phobos, which travels in an approximately circular orbit of radius  $9.4 \times 10^6$  m with a period of 7 hours, 39 minutes. From this information and Newton’s law of gravity, determine the mass of Mars.
3. Communication satellites are placed in circular orbits above the earth’s equator at such a height that they orbit once every 24 hours, and therefore appear to remain fixed in our sky. How far above earth’s surface is such a satellite? Please solve this problem from first principles (not from Kepler’s third law).
4. A projectile is fired vertically from Earth’s surface with an initial speed of 10 km/s. Neglecting air drag, how far above Earth’s surface will it go?
5. An asteroid of mass  $3 \times 10^{15}$  kg is observed to be heading straight toward the earth. At the time of observation, the asteroid is ten million kilometers away and has a speed (relative to the earth) of 8.5 km/s. What is its kinetic energy just before hitting earth? Compare your result to the energy released by a 1 megaton nuclear bomb (about  $4 \times 10^{15}$  J). What forms might this energy take after the asteroid hits earth?
6. Calculate “escape speed” from the moon’s surface, that is, the minimum speed needed for an unpowered projectile to escape rather than falling back.
7. While prospecting for titanium among the asteroids (in order to find raw materials for building more spaceships, so more people can go prospecting for titanium, and so on), you discover a spherical asteroid with a radius of 3 km. You put your spaceship into a circular orbit around this asteroid, at a height of 2 km above the surface, then descend to the surface on your handy jet-pack. Upon arriving at the surface, you discover that the asteroid is made of solid titanium (which has a density of  $4.54 \text{ g/cm}^3$ )! You are overjoyed by this discovery—so overjoyed, that while jumping up and down for joy, you accidentally land on your jet-pack, demolishing it. Fortunately, you have a back-up jet-pack, but unfortunately, that’s exactly where it is: back up on the spaceship. However, all hope is not lost. Because the asteroid’s gravitational pull is so much less than earth’s, you wonder whether you might be able to *jump* up to your spaceship. You know that on earth, while wearing your cumbersome spacesuit, you can jump only 0.5 meters (as measured by your center of mass). Given this information, can you jump to your spaceship? Please explain carefully.
8. You are the commander of the starship Elliptica, orbiting an unexplored planet (mass  $3.0 \times 10^{24}$  kg) in a circular orbit of radius  $1.0 \times 10^7$  m. (a) What is your current orbital speed? (b) The scientists on your crew are having another argument: one wants to

stay in the current orbit, far above the surface, to study the planet's magnetosphere; the other wants to get closer, to search for possible life on the planet's surface. In an attempt to please them both, you give the order to go into an elliptical orbit, with a maximum distance from the planet's center equal to the current orbital radius and a minimum distance equal to exactly half this radius. In order to accomplish this, the ship's engineer will briefly fire the thrusters forward, to decrease your speed. What should be your speed, just after the thrusters are fired (but while you are still at your present location)? (Hint: apply both energy conservation and angular momentum conservation to the two extreme points in the desired orbit.)

9. How do astronauts "weigh" themselves in space? They sit in a chair attached to a large spring, and measure the period of oscillation. (a) If  $M$  is the mass of the astronaut and  $m$  is the effective mass of the chair and the moving end of the spring, show that  $M = (k/4\pi^2)T^2 - m$ , where  $T$  is the period of oscillation and  $k$  is the spring constant. (b) The spring constant of the device used on Skylab was 605.6 N/m, and the period of oscillation of the empty chair was 0.90149 s. Calculate the effective mass of the chair. (c) With an astronaut in the chair, the period of oscillation was 2.08832 s. Calculate the mass of the astronaut.
10. Suppose you have two identical springs, with different masses attached, each oscillating with the same amplitude. The mass for System 1 is twice that of System 2. Discuss how the following quantities differ for the two systems: frequency, period, total energy, maximum speed, maximum kinetic energy.
11. Explain carefully why the period of a pendulum is approximately independent of its amplitude, so long as the amplitude is fairly small. Your explanation should include a discussion of the torque exerted by gravity, and of how this determines the pendulum's average speed. Then explain why the period *does* increase measurably when the amplitude is large.
12. (a) Sketch a graph of the function  $y = e^{-x^2}$ , for values of  $x$  ranging from  $-5$  to  $5$ . (b) Sketch a graph of a function that is the same as in part (a), but displaced to the right by 3 units. (c) Write down a formula for the function that you plotted in part (b), and explain in English why you modified the original formula in the way that you did.

## Study Guide

### Gravity

All objects exert attractive gravitational forces on all other objects. For two point masses  $m_1$  and  $m_2$  separated by a distance  $r$ , the magnitude of the force is

$$|\vec{F}_g| = \frac{Gm_1m_2}{r^2},$$

where  $G = 6.67 \times 10^{-11}$  N-m<sup>2</sup>/kg<sup>2</sup>. A symmetrical sphere such as the earth may be treated as a point mass, as if all the mass were located at the center.

The potential energy function associated with this force is

$$U_g = -\frac{Gm_1m_2}{r},$$

where the “reference point” is taken to be at infinite separation. (This choice is arbitrary, but makes the formula simpler.) The minus sign indicates that the potential energy decreases as the two objects get closer together.

You should be able to solve constrained-motion problems, energy-conservation problems, and angular-momentum-conservation problems involving motion of planets, satellites, and other objects moving under the influence of gravity.

### Oscillatory Motion

For a simple mass-spring system, the differential equation of motion (derived from Newton’s second law) is

$$\frac{d^2x}{dt^2} = -\frac{k_s}{m}x.$$

A solution to this equation is the function

$$x(t) = A \cos \omega t,$$

where  $A$  (the “amplitude”) can be any distance and  $\omega$  (the “angular frequency”) is  $\sqrt{k_s/m}$ . (A sine function with the same angular frequency is also a solution.) The period of oscillation (time to go back and forth once) is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k_s}},$$

and the ordinary frequency is  $f = 1/T = \omega/2\pi$ .

*Many* other systems oscillate in a similar way, and the mathematics is exactly the same. Whenever the equation of motion says that the acceleration is minus some constant times the position, you can conclude that the motion obeys a cosine or sine function with  $\omega$  equal to the square root of that constant. An important special case is a simple pendulum of length  $L$ , whose period is  $2\pi\sqrt{L/g}$ .