

Problem Set 10
(due Wednesday, March 22)

1. Estimate the gravitational force between you and a person sitting 2 m away from you. (You should find that the attraction is negligible. As Einstein once said, “Gravity cannot be held responsible for people falling in love.”)
2. The planet Mars has a satellite (moon), called Phobos, which travels in an approximately circular orbit of radius 9.4×10^6 m with a period of 7 hours, 39 minutes. From this information and Newton’s law of gravity, determine the mass of Mars.
3. Communication satellites are placed in circular orbits above the earth’s equator at such a height that they orbit exactly once every 24 hours, and therefore appear to remain fixed in our sky. How far above earth’s surface is such a satellite? Please solve this problem from first principles (not from Kepler’s third law).
4. A projectile is fired vertically from Earth’s surface with an initial speed of 10 km/s. Neglecting air drag, how far above Earth’s surface will it go?
5. An asteroid of mass 3×10^{15} kg is observed to be heading straight toward the earth. At the time of observation, the asteroid is ten million kilometers away and has a speed (relative to the earth) of 8.5 km/s. What is its kinetic energy just before hitting earth? Compare your result to the energy released by a 1 megaton nuclear bomb (about 4×10^{15} J). What forms might this energy take after the asteroid hits earth?
6. Calculate “escape speed” from the moon’s surface, that is, the minimum speed needed for an unpowered projectile to escape rather than falling back.
7. While prospecting for titanium among the asteroids (in order to find raw materials for building more spaceships, so more people can go prospecting for titanium, and so on), you discover a spherical asteroid with a radius of 3 km. You put your spaceship into a circular orbit around this asteroid, at a height of 2 km above the surface, then descend to the surface on your handy jet-pack. Upon arriving at the surface, you discover that the asteroid is made of solid titanium (which has a density of 4.54 g/cm^3)! You are overjoyed by this discovery—so overjoyed, that while jumping up and down for joy, you accidentally land on your jet-pack, demolishing it. Fortunately, you have a back-up jet-pack, but unfortunately, that’s exactly where it is: back up on the spaceship. However, all hope is not lost. Because the asteroid’s gravitational pull is so much less than earth’s, you wonder whether you might be able to *jump* up to your spaceship. You know that on earth, while wearing your cumbersome spacesuit, you can jump only 0.5 meters (as measured by your center of mass). Given this information, can you jump to your spaceship? Please explain carefully.
8. You are the commander of the starship Elliptica, orbiting an unexplored planet (mass 3.0×10^{24} kg) in a circular orbit of radius 1.0×10^7 m. (a) What is your current orbital speed? (b) The scientists on your crew are having another argument: one wants to

stay in the current orbit, far above the surface, to study the planet's magnetosphere; the other wants to get closer, to search for possible life on the planet's surface. In an attempt to please them both, you give the order to go into an elliptical orbit, with a maximum distance from the planet's center equal to the current orbital radius and a minimum distance equal to exactly half this radius. In order to accomplish this, the ship's engineer will briefly fire the thrusters forward, to decrease your speed. What should be your speed, just after the thrusters are fired (but while you are still at your present location)? (Hint: apply all applicable conservation laws to the two extreme points in the desired orbit.)

9. Imagine a large window measuring 2 m by 3 m. Inside the window, the pressure is held constant at 1 bar, while outside, during a storm, the pressure momentarily drops to 0.96 bar. What net force acts on the window (not counting the frame that we hope will hold it in place)?
10. Estimate the pressure you exert on the ground when standing (a) in normal shoes; (b) in ice skates; (c) in snowshoes. Please express your answers both in pascals and in bars (atmospheres).
11. Estimate the total weight of the air directly above your head.

Study Guide

All objects exert attractive gravitational forces on all other objects. For two point masses m_1 and m_2 separated by a distance r , the magnitude of the force is

$$|\vec{F}_g| = \frac{Gm_1m_2}{r^2},$$

where $G = 6.67 \times 10^{-11}$ N-m²/kg². A symmetrical sphere such as the earth may be treated as a point mass, as if all the mass were located at the center.

The potential energy function associated with this force is

$$U_g = -\frac{Gm_1m_2}{r},$$

where the "reference point" is taken to be at infinite separation. (This choice is arbitrary, but makes the formula simpler.) The minus sign indicates that the potential energy decreases as the two objects get closer together.

You should be able to solve constrained-motion problems, energy-conservation problems, and angular-momentum-conservation problems involving motion of planets, satellites, and other objects moving under the influence of gravity.

(A study guide on fluids will be provided with Problem Set 11.)