Computer-Simulated Projectile Motion

Name ______________________

Partners ______________________

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Lab Station ______________________

Date ______________________

Overview:

This experiment is not an experiment. Instead, during this lab session you will use a computer spreadsheet to predict the motion of a projectile with air resistance.

Physics principles:

Velocity and acceleration
Air resistance

Physics in the Real World

By now you've surely noticed that your introductory physics course is full of simplifications. For instance, the homework problems in your textbook always tell you to ignore air resistance. This is because air resistance invalidates all those nice, simple equations that let you predict a projectile's motion in just a few algebraic steps. But physics *does* apply to the real world, where air resistance and other complications make a real difference. We just need to do a lot more calculation to connect theory to experiment.

Fortunately, modern electronic computers can perform extremely lengthy calculations in a negligible amount of time. These days, therefore, computers offer the best avenue toward applying the basic laws of physics to complicated, real-world situations. In this exercise you will use a computer spreadsheet program to predict the motion of a projectile, including the effects of air resistance. This is just one example of a vast discipline called **computational physics**. Once you understand this example, others will hold little mystery.

Approximate Kinematics

The starting point for simulating particle motion is the basic definitions of velocity and acceleration. For linear motion in the y direction, the velocity is

$$v_y = \frac{\Delta y}{\Delta t} = \frac{y_{\text{final}} - y_{\text{initial}}}{\Delta t}. \tag{1}$$

In a computer simulation of the motion, you already know the current value of y and you want to predict the future value. So let's solve this equation for y_{final}:

$$y_{\text{final}} = y_{\text{initial}} + v_y\,\Delta t. \tag{2}$$

Similarly, you can predict the future value of v_y if you know the current value as well as the acceleration:

$$v_{y,\text{final}} = v_{y,\text{initial}} + a_y\,\Delta t. \tag{3}$$

These equations are exact if v_y and a_y are the *average* velocity and acceleration over the time interval in question. Normally, however, you don't know these average quantities in advance; instead you have to settle for the *initial* values. In that case the equations are still valid in the limit where $\Delta t \longrightarrow 0$, and approximately valid as long as Δt is sufficiently small:

$$y_{\text{final}} \approx y_{\text{initial}} + v_{y,\text{initial}}\,\Delta t; \qquad v_{y,\text{final}} \approx v_{y,\text{initial}} + a_{y,\text{initial}}\,\Delta t. \tag{4}$$

By calculating these quantities over and over again, you can gradually simulate the passage of time and the motion of the particle. (This procedure for predicting a particle's motion is known as the **Euler algorithm** (pronounced "oiler"), after the Swiss mathematician Leonhard Euler.) Meanwhile, you can always calculate the current acceleration by summing up the forces that act on the particle:

$$a_y = \frac{\sum F_y}{m}. \tag{5}$$

The Spreadsheet

To simulate particle motion on a spreadsheet, you'll make a large table with colums for time, position, velocity, force, and acceleration. We'll restrict the motion to the vertical direction, with the y axis taken to point upward. Each row of the table will correspond to a particular time, with time increasing as you go downward. Above the table you'll need to define some constants, which can be changed later if desired. An illustration of the top portion of the spreadsheet is shown on the following page.

Go ahead and launch your spreadsheet program, and type in the text for the column headings and other labels. Also enter the values for the four constants in column C.

Enter a 0 in cell A10 for the initial time, then enter a formula in cell A11 that adds the dt value (in cell C7) to obtain the next t value. Use dollar signs ("C7") to create an absolute reference to cell C7 that won't be changed when you copy this formula downward. Copy it downward for several more cells and check that each time is greater than the last by 0.1. Try changing the value of dt in cell C7 and make sure the t values change accordingly; then change dt back to 0.1.

	A	B	C	D	E
1	One-dimensional projectile motion with air resistance				
2	R. Hooke and I. Newton				
3					
4	Constants:	g =	9.8	m/s^2	
5		m =	0.1	kg	
6		b =	0		
7		dt =	0.1	s	
8					
9	t (s)	y (m)	v_y (m/s)	F_y (N)	a_y (m/s^2)
10	0	5	0	-0.98	-9.8
11	0.1	5	-0.98	-0.98	-9.8
12	0.2	4.902	-1.96	-0.98	-9.8

Figure 1: Beginning of a spreadsheet to simulate projectile motion in the y direction.

Enter the initial position and velocity in cells B10 and C10. Use the values shown for now, remembering that you'll be changing these later.

Column D will hold formulas to calculate the force at any time. Eventually this force will include air resistance, but for now just put a formula into cell D10 for the constant force of gravity, calculated from the m and g values above (again, use dollar signs for absolute references to these cells).

Enter a formula in cell E10 to calculate the acceleration from the force, using Newton's second law. Again, use dollar signs for the absolute reference to the mass.

Now that the initial values are in place (row 10 in the spreadsheet), go down to cell B11. Here you want to calculate the object's new position, using its previous position and velocity. According to equation (4), the correct formula is:

$$= \texttt{B10} + \texttt{C10} * \texttt{\$C\$7} \tag{6}$$

This formula takes the previous position and adds on a displacement equal to $v_y\,\Delta t$, where v_y is the previous velocity.

Enter a similar formula in cell C11 to calculate the new velocity from the previous velocity and acceleration.

To fill cells D11 and E11 you should be able to simply copy the formulas from the previous row.

And now that row 11 is complete, you can copy this entire row downward to carry the computation forward in time. That is, the values in row 11 become the new initial conditions for the calculation in row 12; these values then become the initial conditions for the calculation in row 13; and so on, as far down as you like! Add about 20 more rows to the table at this time.

Question: According to your spreadsheet, at about what time does the falling object reach $y = 0$? Check that this answer is *approximately* correct, based on what you already know about freely falling projectiles. (Don't expect exact agreement.) Show your calculations and discuss your result briefly.

Question: Change the value of dt to 0.01 seconds, and add enough rows to the table to calculate the motion down to $y = 0$. When does the table predict that the object will reach $y = 0$? How does changing dt affect the accuracy of the prediction?

The preceding questions illustrate a crucial fact about computer simulations: The results are never exact. Inaccuracies caused by taking dt to be larger than zero are called **truncation errors**. Fortunately, you can make truncation errors smaller by using a smaller value of dt. Unfortunately, when dt is smaller you must also add more rows to the table and the whole computation becomes more cumbersome.

Question: About how small must dt be in order to predict the landing time (when the object reaches $y = 0$) to within 1%? Justify your answer. (From now on, please use a value of dt that is at least this small.)

Air Resistance

Under most conditions, the force of air resistance on an object is proportional to the *square* of the object's speed:

$$|\vec{F}_{\text{air}}| = b|\vec{v}|^2, \tag{7}$$

where b is a constant that depends on the size and shape of the object, and on the density of the air. (The quadratic dependence on speed is due to the fact that a faster object collides with *more* air molecules per second, and also collides *more violently* with each of them.)

For a symmetrical, nonspinning object, the direction of the force of air resistance is always opposite to the object's velocity.

Question: What are the SI units of the constant b in the formula above? (Please also enter these units in cell D6.)

Exercise: Given the formula above for the magnitude of the air force, find a correct formula for the y component of the air force acting on an object that is moving in the y direction. Write your formula in terms of v_y, and be sure to explain how your formula has the correct sign for either sign of v_y. (Hint: You'll need to use an absolute value function in a tricky way.)

After making sure you've answered the previous exercise correctly, modify the formulas in the force column of your spreadsheet to add an air resistance force to the gravity force. The absolute value function is "Abs( )". Notice that the constant b is already in cell C6, but its current value is zero. In the space below, write the new formula that appears in cell D10.

Exercise: Change the value of b in your spreadsheet and observe the results. For what value of b does the landing time equal approximately 2.0 seconds?

Exercise: Modify the constants and initial y value in your spreadsheet to model the falling of a coffee filter, under the same conditions as in last week's experiment. For this exercise you'll need to measure the mass of a coffee filter and adjust the value of b until the predicted motion agrees with your measurements from last week. Write down your values of m and

b, and explain how you made the comparison between your spreadsheet prediction and last week's measurements.

Exercise: Make a graph of velocity vs. time for the predicted motion of a falling coffee filter. Print your graph and attach it to this report.

Exercise: Now modify your constants and initial conditions to model the motion of a baseball thrown vertically upward (by a professional pitcher) at a speed of 40 m/s. The mass of a baseball is 145 g and its b value is approximately 0.001 in SI units. How high does the baseball go before it stops rising? How fast is the ball moving when it returns to ground level? How would its maximum height change if there were no air resistance? (Write down the data to justify each of your answers.)

Exercise: Make a graph of position vs. time for the motion of the rising and falling baseball. Print your graph and attach it to this report.

Exercise: Change all the constants and initial conditions in your spreadsheet, including dt, back to the values shown in Figure 1. Then change the value of b to 0.1 in SI units. Delete all rows in the spreadsheet past $t = 2.0$ seconds. Then print your spreadsheet and attach the printout to this report.

Congratulations! Now that you've completed this assignment, you can call yourself a computational physicist.