Physics 2710 (Schroeder)
fall 2012

Name ______________________

Problem Set 9

(due Wednesday, November 14, 5:00 p.m.)

1. Consider an electron that is in the spin state

$$\psi = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

(a) Show that this spinor is properly normalized.

(b) Express ψ as a linear combination of $\psi_{z\uparrow}$ and $\psi_{z\downarrow}$, identifying the coefficients $c_\uparrow$ and $c_\downarrow$.

(c) Express ψ as a linear combination of $\psi_{x\uparrow}$ and $\psi_{x\downarrow}$, identifying the coefficients $c_\uparrow$ and $c_\downarrow$.

(d) Suppose you were to measure S_z for this electron. What values could you obtain, with what probabilities?

(e) Suppose you were to measure S_x for this electron. What values could you obtain, with what probabilities?

(f) Suppose that you measure S_x and obtain the value $+\hbar/2$. After this measurement, what is the spinor that describes the state of the electron?

2. When a spin-1/2 particle is in a state of definite S_y, a measurement of S_x or S_z is equally likely to return $+\hbar/2$ or $-\hbar/2$. Use this fact to find the S_y eigenspinors, $\psi_{y\uparrow}$ and $\psi_{y\downarrow}$. (Hint: You'll have to use imaginary numbers.) Don't worry about which is which, and remember that the overall phase is ambiguous.

3. Imagine an electron immersed in a uniform magnetic field $\vec{B}$ that points in the z direction. As discussed in Section 9.6, this means that the state $\psi_{z\uparrow}$ acquires an energy $+\mu_B B$, while the state $\psi_{z\downarrow}$ acquires an energy $-\mu_B B$, where μ_B is the Bohr magneton, $e\hbar/2m_e$.

(a) Calculate the Bohr magneton numerically, in J/T and in eV/T.

(b) Suppose that at $t = 0$, this electron is in the state $\psi_{x\uparrow}$. Find the state of the electron at later times, as a function of t. (Hint: First write $\psi(0)$ as a linear combination of the definite-energy states $\psi_{z\uparrow}$ and $\psi_{z\downarrow}$. Then, just as we did with one-dimensional spatial wavefunctions, multiply each definite-energy term by its appropriate wiggle factor.) Simplify your result as much as possible, and discuss what it means.

(c) If the magnetic field strength is one tesla, what is the frequency of the oscillation (called "Larmor precession") that you discovered in part (b)?

4. (T+Z, Problem 9.19.) Consider a hydrogen atom in its ground state, placed in a magnetic field of 0.7 T along the z axis.

(a) What is the energy difference between the spin-up and spin-down states?

(b) An experimenter wishes to excite the atom from the lower to the upper spin level by sending in photons of the appropriate energy. What energy is this? What photon wavelength is needed? What kind of radiation is this (visible, UV, . . .)?

5. (T+Z, Problem 10.3.) Estimate the energy of the innermost electron of lead. Also estimate its most probable radius (i.e., distance from the nucleus). (You can look up the charge of the nucleus in your textbook or on any periodic table.)

6. (T+Z, Problem 11.15.) Draw four energy level diagrams, similar to those in Figure 10.7 (11.7 in the first edition), to illustrate the ground states of boron, fluorine, neon, and sodium.

7. (T+Z, Problem 11.19.) In this problem you will estimate the total energy of a helium atom.

   (a) What would be the total energy of a helium atom (in its ground state) in the approximation that you ignore completely the electrostatic force between the two electrons? (Hint: Just compute the separate energies of the two electrons, and add 'em up.)

   (b) Your answer in part (a) should be negative (indicating that the system is bound), and *too* negative, since you ignored the positive potential energy due to repulsion between the electrons. To estimate this additional potential energy, pretend that the electrons are classical particles in the first Bohr orbit (but remember that $Z = 2$ for the helium nucleus). To minimize their energy, the two electrons will always stay on opposite sides of the orbit from each other. What, then, is the potential energy due to their interaction? Combine this number with your answer to part (a) to obtain the total energy of the two electrons. Compare to the observed value, -79.0 eV.

8. (T+Z, Problem 11.21.) Use the energy level diagram of Figure 10.9 (11.9 in the first edition) to write down the electron configurations (i.e., $1s^2 2s^2 2p^6 \cdots$) of Zn, Br, Xe, At, and Fr.