Physics 2710 (Schroeder) Name ______________________________
fall 2012

Problem Set 8

(due Wednesday, November 7, 5:00 p.m.)

1. In this problem you will find some of the wavefunctions and energies for the "spherical rigid box" potential: $U(r) = 0$ for $r < a$, and $U(r) = \infty$ for $r > a$.

   (a) Consider first the case $\ell = 0$. Show that the solutions $u(r)$ to the radial Schrödinger equation are sine waves, and write down the general formulas for $u(r)$ and $R(r)$, in terms of a quantum number n. Find a formula for the allowed energies in terms of n. Finally, sketch a "cloud-shaped diagram" showing the three-dimensional (or at least, two-dimensional) appearance of these wavefunctions.

   (b) Now consider the case $\ell = 1$. Sketch the "effective potential" for this case, as a function of r, and then draw qualitative sketches of $u(r)$ for the three lowest-energy wavefunctions. Point out how these differ from the $\ell = 0$ solutions. Sketch some cloud-shaped diagrams of these wavefunctions (remembering that there are three possible values of $m = L_z/\hbar$).

   (c) (Extra credit.) Use Mathematica to find the three lowest-energy wavefunctions and the corresponding energies for $\ell = 1$. (Hints: First transform the radial Schrödinger equation to dimensionless units. Because the equation contains a term with r in the denominator, tell Mathematica to start at a small nonzero value of r/a, such as 0.001. For boundary conditions, set $u(r) = 0$ and $u'(r) =$ something small at this point.)

2. Consider the $n = 2$, $\ell = 1$ radial wavefunction for hydrogen. (This is called the "2p orbital".)

   (a) Sketch a graph of this function, that is, $R(r)$. (Don't worry about the vertical scale of the graph. Label the horizontal scale in units of a_B.)

   (b) Sketch a graph of the corresponding function $u(r)$.

   (c) Plug this function $u(r)$ into the radial Schrödinger equation, to show that it is indeed a solution. In the process, find the corresponding energy E.

3. Repeat the previous problem for the $n = 2$, $\ell = 0$ radial wavefunction (called the "2s orbital").

4. Suppose that an electron in a hydrogen atom is in the 2s state, and that you intend to measure the position of this electron.

   (a) At what *point* would you be most likely to find the electron? Justify your answer, by referring to the graphs in the previous problem.

   (b) At what *value of* r would you be most likely to find the electron? Justify your answer.

   (c) What is the probability of finding the electron at $r < 2a_B$, that is, in the inner "bump"? (Hint: There is no need to do a three-dimensional integral; just work with the function $u(r)$.)

5. Write down, separately, the full formulas for each of the four independent, separable hydrogen wavefunctions, $\psi(r, \theta, \phi)$, for $n = 2$. Sketch a "cloud-shaped diagram" showing the rough appearance of each of these wavefunctions in three-dimensions, either using colored pencils to represent phases or simply labeling phases in selected locations ($+1$, $+i$, -1, $-i$, for instance).

6. Draw a cone diagram, as in the previous problem set, showing the spin states of a particle with $s = 1/2$. Repeat for a particle with $s = 3/2$. Draw both diagrams on the same scale, and be as accurate as you can with magnitudes and directions.