Physics 2710 (Schroeder) Name ______________________________
fall 2012

# Problem Set 7

(due Wednesday, October 31, 5:00 p.m.)

1. (T+Z, Problems 8.9, 8.10.)

   (a) Consider a nonrelativistic particle of mass $M$ in a two-dimensional rigid box with widths $a$ (in the $x$ direction) and $b$ (in the $y$ direction). Using arguments similar those given in class, find a general expression for the definite-energy wavefunctions for this system, as well as an expression for the corresponding energies.

   (b) Draw an energy level diagram for this system, showing the lowest six levels and their degeneracies, for the case $b = a/2$.

   (c) Repeat part (b) for the case $b = a/4$. Discuss what would happen in the limit $b \ll a$.

2. A nonrelativistic particle is confined inside a three-dimensional cube-shaped rigid box.

   (a) Draw an energy level diagram for this particle, showing all states with energies below $15 \cdot (h^2/8Ma^2)$. Be sure to show each linearly independent state separately, to indicate the degeneracy of each energy level.

   (b) Does the average number of states per unit energy increase or decrease as $E$ increases?

3. One way to visualize the states of a particle in a three-dimensional cube-shaped box is to plot them as dots on a three-dimensional graph whose axes are $n_x$, $n_y$, and $n_z$. Note that the energy of a state is proportional to $n_x^2+n_y^2+n_z^2$, so $E$ is related to the *distance* from the origin in this graph. Use this picture to estimate the number of independent states with energy less than 1.0 eV available to an electron confined inside a cube that measures one centimeter along each side.

4. Draw a large circle on a sheet of paper, and "plot" the function $e^{im\phi}$, for the case $m = 3$ (corresponding to $L_z = 3\hbar$), as follows: Write numbers around the circle to indicate each angle where the function is equal to 1, where it is equal to $+i$, where it is equal to $-1$, and where it is equal to $-i$. "Plot" a few intermediate values as well, just to make sure you get the idea. (Optionally, you may enjoy making up a color scheme to represent phases of complex numbers, then coloring your circle accordingly.)

5. (T+Z, Problem 8.25.) Draw a "vector model" diagram, similar to figure 9.14 (or 8.14 in the second edition), for the case $\ell = 3$. Be sure to carefully measure both the lengths of the $\vec{L}$ vectors and their $z$ components. Then sketch the corresponding "cone" diagram (as in class), to show the uncertainty in $L_x$ and $L_y$ for these states.

6. Imagine a rigid dumbbell in three-dimensional space, which can rotate about its center of mass. This is an excellent model of the behavior of a diatomic molecule with nonidentical atoms (e.g., CO) under many conditions. (For a diatomic molecule with identical atoms, like $O_2$, there is a slight complication that we'll ignore in this problem.) It is conventional to use the symbol $\vec{J}$, rather than $\vec{L}$, for the angular momentum of a molecule. Classically, then, the energy of this system would be

$$E = \frac{|\vec{J}|^2}{2I},$$

where $I$ is its moment of inertia. In quantum mechanics, this formula still holds, but the values of $|\vec{J}|$ are quantized according to the usual formula ($\sqrt{j(j+1)}\,\hbar$, for nonnegative integers $j$), and each energy level has a degeneracy equal to the number of possible $J_z$ values for the given $|\vec{J}|$. Draw an energy level diagram for this system, showing the four lowest levels and their degeneracies. Label the levels with their $j$ values and energies.

7. The spacing of rotational energy levels in molecules is ordinarily measured by microwave spectroscopy: bombarding the molecule with microwaves and looking at what frequencies are absorbed.

(a) For a CO molecule, the quantity $\hbar^2/2I$ is approximately 0.00024 eV. What microwave frequency would induce a transition from the $j=0$ level to the $j=1$ level? What frequency would induce a transition from the $j=1$ level to the $j=2$ level?

(b) Use the measured value of $\hbar^2/2I$ to calculate the moment of inertia of a CO molecule.

(c) From the moment of inertia and the known atomic masses, calculate the "bond length," or distance between the nuclei, for a CO molecule.