Physics 2710 (Schroeder) Name ______________________
spring 2012

Problem Set 6

(due Wednesday, October 17, 5:00 p.m.)

1. Qualitative wavefunction sketches: Please sketch the indicated wavefunctions on the accompanying worksheet.

2. In this problem you will solve the Schrödinger equation numerically, for the potential energy function $V(x) = \alpha|x|$.

   (a) Before attempting to solve a complicated differential equation, it's always a good idea to rewrite it using dimensionless variables. To do this, show first that the quantity $\ell \equiv (\hbar^2/2m\alpha)^{1/3}$ has units of length. Then define the dimensionless variable $z \equiv x/\ell$ and the dimensionless energy parameter $\eta \equiv 2m\ell^2 E/\hbar^2$. Rewrite the Schrödinger equation in terms of z and η.

   (b) Use *Mathematica* to find the five lowest-energy definite-energy wavefunctions and the corresponding values of η (to four or five significant figures). Turn in a printout showing your work, including plots of the wavefunctions. (Hints: For the lowest-energy wavefunction, try a horizontal range of z values from -6 to 6. You'll have to increase this range for some of the higher excited states. Try an "initial" wavefunction value of some small number like .001, and an initial derivative of zero.)

   (c) Discuss the qualitative features of these wavefunctions, and their energies, in some detail.

   (d) Evaluate the five lowest allowed energies in electron-volts, assuming that the particle in question is an electron and that α equals one electron-volt per Ångstrom (or 10^{10} eV/m).

3. Show that, when $U(x) = \frac{1}{2}k_s x^2$, the function e^{-ax^2} satisfies the Schrödinger equation, provided that the constant a is chosen appropriately. What is a in terms of m and k_s? What is the energy corresponding to this wavefunction?

4. A CO molecule can vibrate with a natural frequency of $6.4 \times 10^{13}\ \text{s}^{-1}$.

   (a) What are the energies (in eV) of the five lowest vibrational states of a CO molecule?

   (b) If a CO molecule is initially in its ground state and you wish to excite it into its first vibrational level, what wavelength of light should you aim at it?

5. In this problem you will analyze the spectrum of molecular nitrogen; please refer to Figure A.11 in the handout. You may assume that all of the transitions are correctly identified in the energy level diagram.

   (a) What is the approximate difference in energy between the upper and lower electronic states, neglecting any vibrational energy (aside from the zero-point energies $\frac{1}{2}hf$)?

   (b) Determine the approximate spacing in energy between the vibrational levels, for both the lower and upper electronic states.

   (c) Repeat part (b) using a different set of spectral lines, to verify that the diagram is consistent.

   (d) For the lower electronic state, what is the classical oscillation frequency f of the vibrational motion?