

Problem Set 1

(due Wednesday, September 5, 5:00 p.m.)

- (Moore, Problem R1B.2, page 17.) Imagine that we define the rear end of a 120-meter-long train as the origin ($x' = 0$) in the train frame, and we define a certain track signal light as the origin ($x = 0$) in the track frame. Imagine that the rear end of the train passes this signal light at $t = t' = 0$, and that the train is moving in the $+x$ direction at a constant speed of 25 m/s. Twelve seconds later, the engineer turns on the train's headlight.
 - Where (at what x' value) does this event occur in the train frame?
 - Where (at what x value) does this event occur in the track frame?
 - Show that your answers to parts (a) and (b) are consistent with the "Galilean transformation equation," $x' = x - \beta t$.
- (Moore, Problem R1B.4, page 17.) In an effort to attract more passengers, Amtrak trains now offer free bowling in a specially constructed "bowling alley" car. Imagine that such a train is traveling at a constant speed of 35 m/s relative to the ground. A bowling ball is hurled by a passenger on the train, in the same direction that the train is traveling. Observers on the ground measure the ball to be traveling at a speed of 42 m/s. What is the speed of the ball with respect to the train? Show that your result is consistent with the "Galilean velocity transformation," $\vec{v}' = \vec{v} - \vec{\beta}$.
- (Moore, Problem R1A.1, page 19.) A person in an elevator drops a ball of mass m from rest, from a height h above the elevator floor. The elevator is moving at a constant speed of β downward with respect to the enclosing building. (Please express all answers in terms of m , g , h , and β , which you can assume to be "known.")
 - How much time passes before the ball hits the elevator floor?
 - During this time, how far does the elevator move?
 - How far does the ball fall in the building frame before it hits the elevator floor?
 - What is the ball's initial speed in the building frame?
 - Use the law of energy conservation (in the building frame) to predict the ball's final speed in the building frame (as it hits the floor).
 - Use the Galilean velocity transformation and the result of part (e) to find the ball's final speed in the elevator frame.
 - Use the result of part (f) to show that the law of conservation of energy holds in the elevator frame (assuming that it holds in the building frame).
- (Moore, Problem R2B.1, page 35.) Practice with SR units:
 - What is the diameter of the earth in seconds?
 - A sign on the highway reads "Speed Limit 6×10^{-8} ," meaning speed in SR units. Translate this to meters per second and to miles per hour.
 - Show that the unit of acceleration in the SR system is s^{-1} . What is 1 s^{-1} as a multiple of g ?

5. (Moore, Problem R2B.3, page 36.) Show that in the SR unit system, mass, momentum, and energy are all measured in kilograms. Imagine a truck with a mass of 25 metric tons (25,000 kg) barreling down a highway at a speed of 59 miles per hour. What is the truck's momentum in kilograms? What is its kinetic energy in kilograms?
6. (Moore, Problem R2B.5, page 36.) Imagine that you send out a light flash at $t = 3.0$ s as registered by your clock, and receive a return reflection showing your kid brother making a silly face at $t = 11.0$ s as registered by your clock. At what time did your brother actually make this silly face? How far is (or at least, was) your brother away from you (in seconds and in kilometers)? Is this far enough away that he can't really be a nuisance?
7. Draw an accurate spacetime diagram showing the worldlines of the following objects. Be sure to use the same size units for seconds of time and seconds of distance, so light signal worldlines are at 45-degree angles.
 - (a) Particle A passes the point $x = 0$ at time $t = 0$ traveling at a constant speed of $2/3$ in the $+x$ direction.
 - (b) Particle B passes the point $x = 3$ s at $t = 0$ traveling at a constant speed of $1/2$ in the $-x$ direction.
 - (c) Particle C starts from rest at $x = 0$ and $t = -2$ s, and increases speed until it runs into a brick wall at $x = 3$ s and $t = 4$ s, whereupon it remains at rest thereafter.
 - (d) A flash of light (D) is emitted from position $x = 4$ s at time $t = 1$ s and then travels in the $-x$ direction.
8. (Moore, Problem R3B.1, page 52.) Two firecrackers A and B are placed at $x' = 0$ and $x' = 100$ ns, respectively, on a train that is moving in the $+x$ direction relative to the ground. According to synchronized clocks on the train, both firecrackers explode simultaneously. Which firecracker explodes first according to synchronized clocks on the ground? Explain carefully, with the aid of a spacetime diagram.
9. (Moore, Problem R3S.4, page 54.) Imagine two clocks, P and Q . Both clocks leave the spatial origin in the Home frame at time $t = 0$; call this Event O . Both clocks move along the $+x$ axis, with clock P originally traveling at a speed of about $4/5$, while Q travels at a speed of about $1/5$. After a while, however, clock P slows down, comes to rest, and then begins to move back toward the origin. A short time later, it collides with the slower clock Q , which has been moving at constant speed the whole time. Call the collision Event A .
 - (a) Draw a qualitatively accurate spacetime diagram of the situation just described, showing the worldlines of both clocks and Events O and A .
 - (b) Assume that clocks P and Q were both synchronized with the clock at the origin of the Home frame when they left the origin. Will P and Q necessarily agree when they collide? Explain.
 - (c) An observer in the Home frame measures the time between events O and A with a pair of synchronized clocks S_1 and S_2 (one at the location of each event). Clocks P and Q also each measure a time between these two events. Which clock(s) measure proper time between the two events? Which clock(s) measure the spacetime interval between the two events? Which clock(s) measure coordinate time between the two events?

10. With respect to the Home frame, two events are separated by 25 ns in time and by 5 meters in space.
- (a) What is the spacetime interval between these two events?
 - (b) Suppose that a clock is moving in such a way that it directly measures the spacetime interval between these two events. How fast is it moving, with respect to the Home frame?
 - (c) Observers in the Other frame measure the same two events to be separated by 30 ns in time. What is the spatial separation of the events in the Other frame?
11. Suppose you wish to travel to the star Arcturus, which is 36 light-years away. Being in a hurry, you'd prefer not to spend more than 10 years of your time on the spaceship during the one-way journey. How fast must your spaceship travel? (Please draw a spacetime diagram showing the worldlines of the earth, Arcturus, and your spaceship. Use the metric equation to determine the unknown quantities.)
12. You are standing next to a railroad when a train passes by you at a speed of $1/2$. According to your wristwatch, the train takes 3.5 seconds to pass. (It is a very long train!) How long does it take the train to pass you, according to observers on the train? (Hint: Rephrase the question in terms of events. Who measures the spacetime interval between the two events?)
13. A rocket is launched from earth ($x = 0$) at time $t = 0$. The rocket travels away from earth in the $+x$ direction at a constant speed of $3/4$. After 800 s as measured in the earth's frame of reference, the rocket comes to a stop. After another 400 s (in earth's frame), the rocket explodes. A burst of light from the explosion travels back to the earth. Let Event O be the departure of the rocket from earth and Event X be the explosion.
- (a) Draw an accurate spacetime diagram showing these events from the viewpoint of the earth's reference frame.
 - (b) What is the coordinate time between Events O and X ?
 - (c) What is the spacetime interval between Events O and X ?
 - (d) What is the time between Events O and X , as measured by a clock on the rocket? (Hint: break up the journey into two pieces, before and after the rocket comes to a stop.)