## **372** Appendix A Elements of Quantum Mechanics

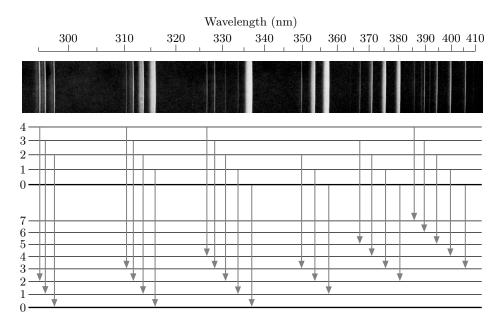


Figure A.11. A portion of the emission spectrum of molecular nitrogen,  $N_2$ . The energy level diagram shows the transitions corresponding to the various spectral lines. All of the lines shown are from transitions between the same pair of electronic states. In either electronic state, however, the molecule can also have one or more "units" of vibrational energy; these numbers are labeled at left. The spectral lines are grouped according to the number of units of vibrational energy gained or lost. The splitting within each group of lines occurs because the vibrational levels are spaced farther apart in one electronic state than in the other. From Gordon M. Barrow, Introduction to Molecular Spectroscopy (McGraw-Hill, New York, 1962). Photo originally provided by J. A. Marquisee.

(e) For the lower electronic state, what is the effective "spring constant" of the bond that holds the two nitrogen atoms together? (Hint: First determine the spring constant for each *half* of the spring, by considering each atom to be oscillating relative to the fixed center of mass. Then think carefully about how the spring constant (force per amount of stretch) of a whole spring is related to the spring constant of each half.)

**Problem A.17.** A *two*-dimensional harmonic oscillator can be considered as a system of two independent one-dimensional oscillators. Consider an isotropic two-dimensional oscillator, for which the natural frequency is the same in both directions. Write a formula for the allowed energies of this system, and draw an energy level diagram showing the degeneracy of each level.

**Problem A.18.** Repeat the previous problem for a *three*-dimensional isotropic oscillator. Find a formula for the number of degenerate states with any given energy.