Electron - Positron Annihilation

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OUTLINE

• Electron-positron storage rings
• Detectors
• Reaction examples
  \[ e^+ e^- \rightarrow e^+ e^- \]
  [Inventory of known particles]
  \[ e^+ e^- \rightarrow \mu^+ \mu^- \]
  \[ e^+ e^- \rightarrow q \bar{q} \]
  \[ e^+ e^- \rightarrow W^+ W^- \]
• The future: Linear colliders
Electron-Positron Colliders

- Stanford
  - 8 GeV
  - 30 GeV
  - 100 GeV
  - 12 GeV
- Ithaca
  - 12 GeV
- Geneva
  - 200 GeV
- Hamburg
  - 11 GeV
  - 47 GeV
- Novosibirsk
  - 12 GeV
- Tokyo
  - 64 GeV
  - 12 GeV
- Beijing
  - 4 GeV
Size \((R)\) and Cost \((\$)\) of an \(e^+e^-\) Storage Ring

\[
\$ = \alpha R + \frac{\beta E^4}{R} \quad (E = \text{beam energy})
\]

Find minimum \$: \quad 0 = \frac{d\$}{dR} = \alpha - \frac{\beta E^4}{R^2}

\[
\Rightarrow R = \sqrt[\alpha]{\beta E^2}, \quad \$ = 2\sqrt[\alpha]{\alpha \beta E^2}
\]

SPEAR: \(E = 8\) GeV, \(R = 40\) m, \(\$ = 5\) million

LEP: \(E = 200\) GeV, \(R = 4.3\) km, \(\$ = 1\) billion
Example 1: \( e^+e^- \longrightarrow e^+e^- \)

- total momentum = 0
- total energy = \( 2E \)
- \( \text{Probability}(E, \theta) = ? \)

\( E \)-dependence follows from dimensional analysis:

\[
\text{density} = \rho_- \underbrace{\ell_-}_{A} \quad \text{density} = \rho_+ \underbrace{\ell_+}_{A}
\]

\[
\text{Probability} = (\rho_- \rho_+ \ell_- \ell_+ A) \times \text{(something with units of length}^2\text{)}
\]

When \( E \gg m_e \), the only relevant length is \( \frac{\hbar}{p} = \frac{\hbar c}{E} \)

\[\implies \text{Probability} \propto \frac{1}{E^2} \]
$e^+ e^- \rightarrow e^+ e^-$ at SPEAR

Augustin, et al., PRL 34, 233 (1975)

$E_{cm} = 4.8$ GeV
Prediction for $e^+e^- \rightarrow e^+e^-$ event rate (H. J. Bhabha, 1935):

$$\left( \frac{\text{event rate}}{\text{rate}} \right) \propto \frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2 E_{\text{cm}}^2} \left[ \frac{1 + \cos^4 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}} - \frac{2 \cos^4 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}} + \frac{1 + \cos^2 \theta}{2} \right]$$

Interpretation of Bhabha’s formula (R. P. Feynman, 1949):

$$= (\text{const.}) \times \left| \begin{array}{c} e^- \quad e^- \quad e^+ \\ e^- \quad e^+ \quad e^+ \\ e^- \quad e^- \quad e^+ \end{array} \right|^2$$

Each diagram represents a complex number that depends on $E$ and $\theta$. Each vertex represents a factor of the electron’s charge, $e = -0.303$. 
Feynman Rules  
(neglecting spin, $\hbar = c = 1$)

Multiply pieces together, integrate over $x$ and $y$ . . .
Higher-Order Diagrams

Also:

More vertices $\implies$ more factors of $e$ $\implies$ smaller value

(any charged particle! ("vacuum polarization")
It Works!


HRS Collaboration (SLAC)

$E_{\text{cm}} = 29$ GeV

$s \sigma / d\Omega$ (GeV$^2$ nb/sr)

- tree-level
- no vacuum polarization
- full radiative corrections

$\cos \theta$

$\theta$
# The Periodic Table

## Particles like the electron (fermions, spin 1/2)

<table>
<thead>
<tr>
<th>Leptons</th>
<th>Quarks (each in 3 “colors”)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$\nu_e$</td>
</tr>
<tr>
<td>0.511 MeV</td>
<td>$&lt; 0.000003$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\nu_\mu$</td>
</tr>
<tr>
<td>106</td>
<td>$&lt; 0.2$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\nu_\tau$</td>
</tr>
<tr>
<td>1777</td>
<td>$&lt; 20$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$-1/3$</td>
</tr>
<tr>
<td>$2/3$</td>
</tr>
</tbody>
</table>

## Particles like the photon (bosons, spin 1)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>photon</td>
<td>“electromagnetism”</td>
</tr>
<tr>
<td>$g$</td>
<td>gluon</td>
<td>“strong interaction”</td>
</tr>
<tr>
<td>$W^\pm$</td>
<td>$Z^0$</td>
<td>“weak interaction”</td>
</tr>
</tbody>
</table>

(Gravity is negligible.)
Example 2: \[ e^+ e^- \rightarrow \mu^+ \mu^- \]

Only one diagram:

\[
2 \left( \frac{e^4}{E^2} \right) (1 + \cos^2 \theta)
\]

(Same as third term in Bhabha formula, provided that \( E \gg m_\mu \).)
Example 3: \( e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons} \)

For \(10 \text{ GeV} < E_{\text{cm}} < 40 \text{ GeV},\)
\[
\frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} = 3 \times \left[ \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right]
\]

\(\uparrow\) \(\uparrow\) \(\uparrow\) \(\uparrow\) \(\uparrow\)

\(d\) \(u\) \(s\) \(c\) \(b\)

colors
\[ R = \frac{\sigma(\text{hadrons})}{\sigma(\mu^+ \mu^-)} \]
2-Jet Hadronic Event

Angular Distribution
MAC detector (SLAC), 1986

\[ E_{cm} = 29 \text{ GeV} \]

- Dotted line: photon diagram only
- Solid line: including Z diagram
Example 3(b): \[ e^+ e^- \rightarrow Z^0 \rightarrow q\bar{q} \]

Higher-order diagrams turn $\infty$ into smooth resonance curve.
Example 4: \( e^+ e^- \rightarrow W^+ W^- \)

- Requires \( E_{cm} > 2m_W = 160 \text{ GeV} \)
LEP Preliminary

08/07/2001

σ_{WW} [pb]

E_{cm} [GeV]

- Full theory
- no ZWW vertex
- only ν_ε exchange
• Direct measurement of $m_W$ can be compared to indirect measurements:

\[
\begin{align*}
\mu^- & \quad u & \quad \nu_\mu \\
\nu_\mu & \quad d & \quad W
\end{align*}
\]

\[
\begin{align*}
\nu_\mu & \quad \bar{\nu}_e \\
\mu^- & \quad e^- & \quad W
\end{align*}
\]

• Corrections from higher-order diagrams must be included,

especially:

\[
\begin{align*}
t & \quad b
\end{align*}
\]

• Results disagree, typically by $\sim 1\%$!

• Simplest solution: new spin-0 “Higgs” particle, $m_h \lesssim 200$ GeV

(Also needed to avoid nonsensical predictions at $E \gtrsim 1000$ GeV)
Looking for the Higgs Particle(s)

- Tevatron (Fermilab, Chicago): $p\bar{p}$, $E_{\text{cm}} = 2000$ GeV
- Large Hadron Collider (CERN, Geneva): $pp$, $E_{\text{cm}} = 14,000$ GeV, 2007
  Discovery likely! Detailed study difficult.
- $e^+e^-$ storage ring, $E_{\text{cm}} = 500+$ GeV? Too big, too expensive.
- $e^+e^-$ linear collider
The Next Linear Collider

$e^+ - 20 - 30 \text{ km} - e^-$

Diagram showing the layout of a linear collider with various components such as accelerators, detectors, and injectors labeled with energy levels and distances. The text describes the collider's components and their functions.
Suggested Reading

• Feynman, *QED: The Strange Theory of Light and Matter*

• Barnett, et al., *The Charm of Strange Quarks*

• Riordan, *The Hunting of the Quark*

• ParticleAdventure.org

• physics.weber.edu/schroeder/feynman/