

Problem Set 3

(due Friday, September 20, 4:00 pm)

1. Find a formula for the electric field at a distance z above the center of a horizontal circular loop of radius r that carries a uniform linear charge density λ . Check that your answer has the expected behavior at $z = 0$ and when $z \gg r$. (Examine the latter limit in more detail than just saying the field goes to zero.)
2. Find the electric field at a distance z above the center of a horizontal circular disk of radius R that carries a uniform surface charge density σ . (You can either do this calculation from scratch, or use the result of the previous problem to simplify it somewhat.) Check that your answer has the expected behavior when $z \gg R$. Also simplify your formula in the limit $z \ll R$, and discuss this limit briefly.
3. In this problem you will use Mathematica to plot the electrostatic fields of several source charge distributions. Please turn in printouts showing your final code and plots, but not your preliminary attempts or other scratch work.
 - (a) As a warm-up, plot the electric field of a single positive point charge q , located at the origin. For simplicity, use units in which the constant $q/(4\pi\epsilon_0)$ is equal to 1. Plot a two-dimensional slice through the center of the region surrounding the point charge, using `VectorPlot`. The scale of the plot is arbitrary in this case, but please center the plot at the origin and be sure to use the same scale for both axes. Because the field vectors become infinitely large at the origin, you'll probably need to use the `VectorScale` option to omit the largest vectors.
 - (b) Next plot the field of a *dipole*, consisting of a positive charge q and a negative charge $-q$. Place the charges on the z axis, one unit above and one unit below the origin, and plot a two-dimensional slice along the xz plane, again using `VectorPlot`.
 - (c) Now let the source be a uniform line segment of positive charge, with length 2 (in arbitrary units) and total charge q , modeled as a row of equally spaced point charges strung out along the x axis between -1 and 1 . Again the plot should show a slice along the xz plane. Use Mathematica's `Sum` function to add up the fields of all these point charges. Please define a variable `n` to represent the total number of point charges in the row, and write the rest of your code in terms of `n` so you can change the value of `n` in just one place without modifying the rest. Start with small values of `n`, then increase `n` until the plot no longer changes. As a check, calculate the field strength numerically at the point $z = 1$ on the z axis, and compare to the formula derived in your textbook, $\mathbf{E} = (1/(4\pi\epsilon_0)) 2\lambda L \hat{\mathbf{z}} / (z\sqrt{z^2 + L^2})$. What are the components of the field at the point $x = z = 1$ (and $y = 0$)?
4. Suppose that the electric field in some region is $\mathbf{E} = kr^3\hat{\mathbf{r}}$, where k is a constant. (a) Find a formula for the charge density ρ in terms of k and r . (b) Find a formula for the total charge contained in a sphere of radius R , centered at the origin. Do this in

two different ways: once using your answer to (a), and once using the integral version of Gauss's law.

5. Use Gauss's law to find a formula for the electric field inside a uniformly charged solid sphere with radius R and charge density ρ . Be sure to state explicitly how you are using symmetry to simplify the flux integral and solve for $|\vec{E}|$. Sketch a graph the magnitude of \vec{E} (on the vertical axis) as a function of r (on the horizontal axis), showing the regions both inside and outside the sphere.
6. Use Gauss's law to find the electric field a distance s away from an infinitely long, straight wire that carries a uniform charge density λ . Be sure to state explicitly how you are using symmetry to simplify the flux integral and solve for $|\vec{E}|$. Compare to the result that we derived in class using Coulomb's law.