Problem Set 11
(due Wednesday, December 5, 12:30 pm)

1. Solar energy is becoming more affordable and popular, but has the major drawback that at any given location, the sun isn’t always shining. One way around this problem is to store energy in batteries that are charged during the day and discharged at night, but batteries have their own drawbacks. Consider, therefore, the possibility of storing energy in the magnetic field of a superconducting solenoidal coil. About how large would such a coil have to be, to store enough energy to power a typical U.S. home for one night? Would it fit in your basement? Feel free to use any reasonable estimates for the total energy that needs to be stored and for the maximum magnetic field that can be attained, but state your estimates clearly and explain how you made them. Feel free to look up some numbers to test whether your estimates are reasonable.

2. A fat cylindrical wire, radius $a$, carries a constant current $I$, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as the surfaces on each side of the gap become charged due to the current flow. Find the electric and magnetic fields in the gap, as functions of time and of the distance $s$ away from the axis. (Assume that the charge on the surfaces is zero at $t = 0$.) Sketch the magnetic field in the $s \phi$ plane, as viewed from the direction of the positively charged surface (so the current points into the page).

3. Consider the electric and magnetic fields

$$E(r, t) = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \theta(r - vt) \hat{r}, \quad B(r, t) = 0,$$

where $\theta(x)$ is the step function, equal to zero when its argument is negative and 1 when its argument is positive. Show that these fields satisfy all four of Maxwell’s equations, and determine $\rho$ and $J$. Describe the physical situation that gives rise to these fields.

4. Consider the charging capacitor of Problem 2, above.

(a) Find the energy density $u_{\text{em}}$ and the Poynting vector $S$ in the gap. Note especially the direction of $S$. Check that the continuity equation for energy is satisfied.

(b) Determine the total energy in the gap as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap, that is, that Poynting’s theorem is satisfied. (In this case $W = 0$, because there is no charge in the gap.)

5. Find the fields, and the charge and current distributions, corresponding to the potentials

$$V(r, t) = 0, \quad A(r, t) = -\frac{1}{4\pi \varepsilon_0} \frac{qt}{r^2} \hat{r}.$$

Then use the gauge function $\lambda = -(1/4\pi \varepsilon_0)(qt/r)$ to transform these potentials, and comment on the result.
6. For your final writing exercise this semester, please type up a formal presentation of
one of the following (your choice!):

- Problem Set 10, Problem 4, and its solution. This problem concerns a simple electro-
magnetic generator. If you choose this problem, be sure to include enough diagrams
to make the solution clear. (You may either draw the diagrams neatly by hand or use
a computer drawing program.) Also please include at least one numerical example,
and some discussion of what it would take to make a practical generator.

- Problem Set 10, Problem 5, and its solution. This problem concerns the electric field
around a solenoidal electromagnet with a time-dependent current. If you choose this
problem, be sure to include the sketches of the scenario and of your results. (You
may either draw the diagrams neatly by hand or use a computer drawing program.)
Also please include a numerical example and discuss the numbers briefly.

- Problem Set 10, Problem 8, and its solution. This problem concerns the energy
associated with a solenoidal electromagnet. If you choose this problem, please also
include at least one numerical example, such as that in Problem 1 above.

- Problem Set 11, Problems 2 and 4, and their solutions. These problems concern
the electric field, displacement current, and resulting magnetic field associated with
a narrow gap in a fat current-carrying wire. If you choose these problems, be sure
to include sketches of the scenario and of the fields. (You may either draw these
diagrams neatly by hand, or use a computer drawing program.) Also please work out
a numerical example and discuss the numbers briefly.

- Explain how the idea that “the boundary of a boundary is zero” is related to con-
servation of electric charge by way of Maxwell’s equations. This is a topic that I’ve
discussed in class; it may especially interest you if you enjoy abstract mathematics.

- Another topic of your choice, with permission. If you would like to write up a formal
presentation of some other problem, or of some other topic in electromagnetic theory,
then please run it by me and I will let you know whether it is suitable for this
assignment.

Regardless of your choice, please put extra effort into writing an introductory para-
graph to motivate your topic. This paragraph should explain why the topic or problem
is interesting and provide any other important context. Elsewhere, try to add insight
and interpretation where appropriate, rather than presenting only a dry mathe-
matical calculation. Use your best English spelling, punctuation, grammar, word choice,
and so on. Present mathematical equations with correct typesetting and punctuation,
incorporating each equation into a grammatically correct sentence.

As always, the target audience for this assignment is a physics student who is taking
the same course but who has not yet worked the problem or seen the specific ideas
that you’re writing about. Try to present everything at an appropriate level of detail
for such a student.

The target length of this writing assignment is about three double-spaced pages. If
you find yourself writing a great deal more than that, then please think carefully about
which details you can reasonably omit.