Electromagnetic Theory
Fall 2019

Problem Set 1

(Items 1 and 2 due Friday, August 30, 4:00 pm;
rest due Friday, September 6, 4:00 pm)

1. Stop by my office for 10 minutes this week to sit down and tell me about yourself. I'd like to know more about your academic interests, but I'd also like to know something about your family, hobbies, and so on—whatever you're willing to share. Besides my official office hours, I'll be in my office most of Monday, Tuesday, and Thursday afternoons.

2. Work through the accompanying tutorial, "Plotting functions of more than one variable with Mathematica". Take the time to type each example statement (and option) into Mathematica, and check the results. Be sure to try plenty of variations, so you can see the effect of each bit of code and understand how to modify your plots to suit a wide variety of circumstances. To save paper, don't print or turn in most of this work. Please do turn in, on no more than two printed pages, one contour plot of your choice that uses a variety of non-default options (including a non-default `ColorFunction` other than those used in the tutorial), and one vector plot of your choice that uses a variety of non-default options (including a non-default `VectorColorFunction` and a non-default `VectorScale`). Your printout should also include all of the Mathematica code that you used to produce these two plots. Feel free to use the color printer in room TY 127 (but please don't disturb any classes going on in that room).

3. Not every arrangement of English words is a grammatically correct sentence, and not every arrangement of mathematical symbols is a meaningful expression. Which of the following are meaningful (or if you like, "grammatically correct", whether or not they are true), and which are technically meaningless (whether or not you think you can guess what the writer meant)? (a) $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}| + |\mathbf{B}|$; (b) $\mathbf{E} = (1/4\pi\epsilon_0)q/r^2$; (c) $\mathbf{A} > \mathbf{B} + \mathbf{C}$; (d) $\mathbf{A} = \mathbf{B} \times (\mathbf{C} \cdot \mathbf{D})$; (e) $F_x = ma_y$. Please assume that bold symbols stand for vectors and subscripts indicate vector components, as usual. Be sure to explain each of your answers. (In your hand-written work, please use arrows over symbols, rather than boldface, to indicate vectors.)

4. Prove the **BAC-CAB** rule by writing out both sides in component form.

   General note on proofs: The word "prove" (or "show" or something similar) means that you must make your logic clear. Merely performing the correct algebraic manipulations isn't enough; you'll also need to convey the logical structure of your proof using English words. Whatever you do, *don't* start by writing down the equation you're trying to prove and then manipulating it until you get $0 = 0$ or something else that's obviously true. That's like "proving" that all WSU students are taking Physics 3510, by pointing out that this would imply that *you* are taking Physics 3510, which we know is true!

5. Let $\mathbf{r} = 2\hat{\mathbf{x}} + 5\hat{\mathbf{y}}$, let $\mathbf{r}' = 3\hat{\mathbf{x}} - \hat{\mathbf{y}}$, and let $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$. (You may assume that all numerical values are in some suitable distance unit such as centimeters.)

   (a) Write $\boldsymbol{\mathscr{r}}$ in terms of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$.

   (b) Compute the magnitudes of all three of these vectors, in decimal approximation, from their components using the Pythagorean theorem. Write each result clearly as an equation, using correct notation.

   (c) Carefully draw these three vectors on a single sketch, showing how they are related. Use a ruler to check your answers to part (b).

   (d) Use trigonometry to compute the angles that $\mathbf{r}$ and $\mathbf{r}'$ make with the x axis, in decimal approximation.

   (e) Compute $\mathscr{r}$ using the law of cosines, and check that you get the same result as in part (b).

   (f) Explain why it would be tempting but very wrong to write $\mathscr{r} = r - r'$.

6. Prove that a two-dimensional rotation of coordinates,

$$\begin{pmatrix} \overline{A}_x \\ \overline{A}_y \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix},$$

   leaves the dot-product of any two vectors $\mathbf{A}$ and $\mathbf{B}$ unchanged.

7. Sketch each of the following vector functions in the xy plane, by drawing enough vectors to show the behavior (in all four quadrants). You may draw the vectors at an arbitrary scale, so long as the scale is consistent from one vector to another within each sketch. (You may optionally wish to use Mathematica to check the accuracy of your sketches, but be sure to sketch each function by hand *first*!)

   (a) $\mathbf{v} = x\hat{\mathbf{x}}$

   (b) $\mathbf{v} = y\hat{\mathbf{x}}$

   (c) $\mathbf{v} = \mathbf{r}$

   (d) $\mathbf{v} = (y\hat{\mathbf{x}} - x\hat{\mathbf{y}})/r^2$

8. Looking at your sketches in Problem 7, determine in each case whether the divergence and curl appear to be positive, negative, or zero. (For a two-dimensional vector function of x and y, the curl is simply a number, given by the same formula as for the z component in three dimensions.) Make these determinations graphically and explain your reasoning briefly in each case, showing which arrows on the sketches you used for the determinations. *After* you have made these graphical determinations, use the formulas to *calculate* the divergence and curl of each function. Are there any surprises?

9. Let $\boldsymbol{\mathscr{r}}$ be the separation vector from the arbitrary point (x', y', z') to the arbitrary point (x, y, z), that is, $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$. Also let $\mathscr{r}$ be the magnitude of this vector, that is, $\mathscr{r} = |\boldsymbol{\mathscr{r}}|$. Prove that (a) $\nabla(\mathscr{r}^2) = 2\boldsymbol{\mathscr{r}}$ and (b) $\nabla(1/\mathscr{r}) = -\hat{\boldsymbol{\mathscr{r}}}/\mathscr{r}^2$. Use rectangular coordinates for all calculations.

10. Sketch a two-dimensional slice of the vector function $\mathbf{v} = \hat{\mathbf{r}}/r^2$, and compute its divergence (in three dimensions), using rectangular coordinates. The answer may surprise you; can you explain it?