## Mechanical Energy

Mechanics is the branch of physics that deals with the motion of objects and the forces that affect that motion. Mechanical energy is similarly any form of energy that's directly associated with motion or with a force. Kinetic energy is one form of mechanical energy. In this course we'll also deal with two other types of mechanical energy: gravitational energy, associated with the force of gravity, and elastic energy, associated with the force exerted by a spring or some other object that is stretched or compressed. In this chapter I'll introduce the formulas for all three types of mechanical energy, starting with gravitational energy.

## Gravitational Energy

An object's gravitational energy depends on how high it is, and also on its weight. Specifically, the gravitational energy is the product of weight times height:

$$
\begin{equation*}
\text { Gravitational energy }=(\text { weight }) \times(\text { height }) . \tag{2.1}
\end{equation*}
$$

For example, if you lift a brick two feet off the ground, you've given it twice as much gravitational energy as if you lift it only one foot, because of the greater height. On the other hand, a brick has more gravitational energy than a marble lifted to the same height, because of the brick's greater weight.

Weight, in the scientific sense of the word, is a measure of the force that gravity exerts on an object, pulling it downward. Equivalently, the weight of an object is the amount of force that you must exert to hold the object up, balancing the downward force of gravity. Weight is not the same thing as mass, which is a measure of the amount of "stuff" in an object. If you were suddenly transported to the moon, where gravity is six times weaker than on earth, your weight would be six times less, even though your mass would be unchanged. In interstellar space, far away from earth, moon, and all other gravitating bodies, you would be essentially weightless. Your weight even varies slightly from place to place on earth: more at sea level, less on a mountain top or in a cruising jet airplane. Unless you're an astronaut, though, the variations in your weight as you move from place to place are much less than one percent.

In the official, internationally accepted scientific system of units, an object's height is measured in meters (abbreviated m). One meter is approximately 39.4 inches, or a little over three feet. The official unit of mass is the kilogram $(\mathrm{kg})$, which is the mass of a liter (a little over a quart) of water, or about 2.2 pounds. The official unit of weight, or of any other force, is much less familiar: it is called
the newton (N), after Sir Isaac Newton. One newton is a rather small amount of force, roughly the weight of a small apple (near earth's surface).

The reason why people confuse weight with mass is that at any given location, the weight of an object is directly proportional to its mass. More massive objects are also more weighty, because gravity pulls more strongly on them. In fact, there is a very simple formula for weight in terms of mass:

$$
\begin{equation*}
\text { weight }=(\text { mass }) \times g \tag{2.2}
\end{equation*}
$$

where $g$ is the standard symbol for the local gravitational constant, a measure of the intrinsic strength of gravity at your location. Near earth's surface, the numerical value of $g$ is

$$
\begin{equation*}
g=9.8 \mathrm{~N} / \mathrm{kg} \quad \text { (near earth's surface) } \tag{2.3}
\end{equation*}
$$

implying that a one-kilogram object has a weight of 9.8 newtons (see Figure 2.1). The precise value of $g$ varies from place to place, but again, unless you're an astronaut, those variations are always less than one percent. For many purposes, we can even round off the value of $g$ to $10 \mathrm{~N} / \mathrm{kg}$.


Figure 2.1. This spring scale measures the force being exerted to hold up the chunk of iron. Because this force just balances the downward pull of gravity, it is equal to the weight of the iron. The weight of this one-kilogram chunk of iron is 9.8 newtons.

Using formula 2.2 for weight, we can write equation 2.1 as

$$
\begin{equation*}
\text { Gravitational energy }=(\text { mass }) \times g \times(\text { height }) \tag{2.4}
\end{equation*}
$$

Or, in symbolic notation,

$$
\begin{equation*}
E_{g}=m g h, \tag{2.5}
\end{equation*}
$$

where $m$ stands for mass and $h$ stands for height.
For example, imagine a brick whose mass is 2 kg . The weight of this brick (the force of gravity on it) would be

$$
\begin{equation*}
\text { weight }=m g=(2 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{k} \mathrm{~g})=19.6 \mathrm{~N} \approx 20 \mathrm{~N} . \tag{2.6}
\end{equation*}
$$

And if you lift this brick two meters off the ground, you've given it a gravitational energy of

$$
\begin{equation*}
E_{g}=m g h=(2 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})(2 \mathrm{~m})=39.2 \mathrm{~N} \cdot \mathrm{~m}, \tag{2.7}
\end{equation*}
$$

or about 40 newton-meters. The "newton-meter" is apparently a unit of energy, and in fact, it is the same thing as the joule, the official unit of energy introduced in the previous chapter. Thus, the gravitational energy that you've given the brick is roughly 40 joules.

You may be wondering about the "height" that enters the formula for gravitational energy: Height above what? Good question. The answer is, above any "reference level" you like, so long as you're consistent. The most convenient reference level is usually the floor or the ground (provided it's horizontal), but you could just as well use a tabletop or the ceiling or sea level or any other convenient elevation. Once you pick a reference level for calculating gravitational energy, however, you must continue to use the same reference level throughout your analysis. For instance, when you lift a brick from the floor to the table, you can't compute its initial gravitational energy with respect to the floor but its final gravitational energy with respect to the table, and conclude that both are zero so it hasn't gained any energy. The amount of energy gained is unambiguously positive, and in fact, will come out the same no matter what (consistent) reference level you choose.

If an object is below your chosen reference level, we say that its height is a negative number, and therefore its gravitational energy is negative. There's nothing wrong with this, although it's usually more convenient to put the reference level low enough that all gravitational energies come out positive (or zero).

Notice from the gravitational energy formula that a small mass, if lifted to a great height, can have just as much gravitational energy as a larger mass lifted to a lesser height. For example, lifting a single brick two meters off the ground takes just as much energy as lifting two bricks one meter off the ground. This is the basic principle of several types of "simple machines" including levers, compound pulleys, and hydraulic lifts. Each of these devices uses a smaller weight (or some other force) moving a larger distance to lift a larger weight by a smaller distance (see Figure 2.2).


Figure 2.2. Using a lever or a compound pulley as shown, you can raise a weight by a certain distance by lowering half the weight by twice the distance. In either case, no outside effort is required because there is no net change in gravitational energy.

Exercise 2.1. One kilogram of mass equals 2.2 pounds. Calculate your mass in kilograms. Then calculate your weight (on earth) in newtons.

Exercise 2.2. How would your answers to the previous exercise differ if you were standing on the moon?
Exercise 2.3. I claimed above that a small apple has a weight of about one newton (near earth's surface). What, then, is the mass of such an apple, in kilograms?
Exercise 2.4. Imagine that you are in interstellar space where there is no gravity, and therefore everything is weightless. How could you measure the mass of an object, or tell whether one object is more massive than another, under these conditions?

Exercise 2.5. Suppose that you hike from the WSU campus up to the summit of Mt. Ogden, an elevation gain of about 5000 feet. How much gravitational energy have you gained? Please express your answer in joules and also in kilocalories. (Hint: Convert the elevation gain to meters before plugging it into the formula for gravitational energy.)

Exercise 2.6. A $5-\mathrm{kg}$ bag of groceries sits on the counter top, one meter above the floor. You then lift this bag onto a high shelf, a meter above the counter top. Calculate the gravitational energy of the bag of groceries both before and after you lift it, and also the amount of gravitational energy that it gains during the process, taking the floor as your reference level. Then calculate the same three quantities taking the counter top as your reference level. Finally, recalculate all three quantities taking the shelf as your reference level. Comment on the results.

Exercise 2.7. In a hydroelectric dam, the gravitational energy of the water is converted into electrical energy as the water falls. Consider just one cubic meter $(1000 \mathrm{~kg})$ of water that falls a distance of 500 ft . Assuming that the energy conversion is $100 \%$ efficient, how much electrical energy can be obtained as it falls? Please express your answer both in joules and in kilowatt-hours.
Exercise 2.8. A five-foot plank is used as a lever, with the fulcrum one foot from one end. How much force must you exert on the long end of the lever, in order to lift a $30-\mathrm{kg}$ child standing on the short end?

Exercise 2.9. In a popular trick, a child holds a basketball (mass 600 g ) half a meter off the ground, with a tennis ball (mass 60 g ) resting on top of it. The child then lets go, so the two balls fall together. Suppose that, when they hit the ground, all of the energy of both balls is transferred to the tennis ball, which then shoots vertically into the air. How high will it go?

Exercise 2.10. On August 12, 1973, the author's friend Jock Glidden set a record by ascending the Grand Teton in two hours, 29 minutes. The elevation gain during the ascent was 7000 feet, and the mass of Glidden plus his gear was 60 kg . From this information, estimate Glidden's average mechanical power output, in horsepower.

## Kinetic Energy

The kinetic energy of an object depends on how fast it's moving and also on its mass. The precise formula for kinetic energy in terms of speed and mass is not easy to guess, however. As it turns out, the correct formula is

$$
\begin{equation*}
\text { Kinetic energy }=\frac{1}{2} \times(\text { mass }) \times(\text { speed })^{2} \tag{2.8}
\end{equation*}
$$

or in symbols,

$$
\begin{equation*}
E_{k}=\frac{1}{2} m v^{2} \tag{2.9}
\end{equation*}
$$

where $E_{k}$ represents kinetic energy and $v$ represents speed (or velocity). Let me first show how to use this formula, and then explain how we know that it's correct.

In official scientific units, mass is measured in kilograms and speed is measured in meters per second (m/s). Consider, for instance, a baseball with a mass of 0.15 kg , thrown at a speed of $20 \mathrm{~m} / \mathrm{s}$. The baseball's kinetic energy while it's in motion is

$$
\begin{equation*}
E_{k}=\frac{1}{2}(0.15 \mathrm{~kg})(20 \mathrm{~m} / \mathrm{s})^{2}=30 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}} . \tag{2.10}
\end{equation*}
$$

Numerically, the kinetic energy is 30 , but the units are quite awkward: kilogram meters squared per second squared. Conveniently, however, a $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}$ turns out to be exactly the same as a newton-meter, that is, a joule. How is this possible? Well, I never told you why the unit of force, the newton, was chosen to be the amount that it is. As it turns out, the size of the newton of force has been chosen so that a newton-meter of energy is the same amount as a $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}$ :

$$
\begin{equation*}
1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}=1 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}} \tag{2.11}
\end{equation*}
$$

Our baseball's kinetic energy, therefore, is simply 30 joules.
But why formula 2.9 ? Specifically, why must we square the speed, and why must we multiply by one-half? The answer is that if we used any formula other than this one, energy would not be conserved.

Imagine dropping a heavy ball from the top of a ladder (see Figure 2.3). As the ball falls, its gravitational energy gets converted into kinetic energy. (No other forms of energy are involved during the fall, because the ball never builds up enough speed for air resistance - which would create thermal energy - to become significant.) If energy is to be conserved, then each joule of gravitational energy lost must show up as exactly one joule of kinetic energy gained.

Suppose that the ball's mass is 200 grams (about half a pound). Then after it has fallen one meter, the gravitational energy lost would be

$$
\begin{equation*}
m g h=(0.2 \mathrm{~kg})(9.8 \mathrm{~N} / \mathrm{kg})(1 \mathrm{~m})=1.96 \mathrm{~J} . \tag{2.12}
\end{equation*}
$$

At the one-meter point, therefore, the ball should have exactly 1.96 joules of kinetic energy. To see if this is correct, we must know how fast it's going at this point. The


Figure 2.3. This composite image was made by combining frames taken with a video camera. The time interval between successive images of the falling ball is $1 / 20$ second. At right is an enlargment of a part of the photo, with calculations of the ball's speed near the one-meter and two-meter marks.
illustration above shows how to calculate the ball's speed, by dividing the distance traveled during a short time interval by the time elapsed. At the one-meter point, the ball's speed is approximately $4.4 \mathrm{~m} / \mathrm{s}$. Therefore, according to formula 2.9 , its kinetic energy is

$$
\begin{equation*}
E_{k}=\frac{1}{2} m v^{2}=\frac{1}{2}(0.2 \mathrm{~kg})(4.4 \mathrm{~m} / \mathrm{s})^{2}=1.94 \mathrm{~J}, \tag{2.13}
\end{equation*}
$$

just as expected (given the limited accuracy of the measurements).
To make sure that this agreement isn't just a coincidence, we'd better check energy conservation at another point in the ball's fall. After it has fallen two meters, it has lost twice as much gravitational energy, or 3.92 J . The measured speed at the two-meter point is about $6.3 \mathrm{~m} / \mathrm{s}$, so its kinetic energy has now become

$$
\begin{equation*}
E_{k}=\frac{1}{2} m v^{2}=\frac{1}{2}(0.2 \mathrm{~kg})(6.3 \mathrm{~m} / \mathrm{s})^{2}=3.97 \mathrm{~J}, \tag{2.14}
\end{equation*}
$$

correct again (within our range of uncertainty).
Notice that falling twice as far does not double the ball's speed. If the formula for kinetic energy involved speed to the first power (rather than squared), then
the kinetic energy wouldn't double either, and we couldn't possibly find that the kinetic energy gained is equal to the gravitational energy lost at both points. Using (speed) ${ }^{2}$ in the energy formula solves this problem, though: even though the speed itself doesn't double, the square of the speed does, and so does the kinetic energy.

Before going on to another example, let me point out one more fact about the freely falling ball. Suppose it were twice as massive. Then the gravitational energy it loses while falling one meter would be twice as much. But it wouldn't have to pick up any more speed during the fall, because even at the same speed, its doubled mass would give it twice as much kinetic energy. In fact, any dropped object should be moving at approximately $4.4 \mathrm{~m} / \mathrm{s}$ after falling one meter, at $6.3 \mathrm{~m} / \mathrm{s}$ after falling two meters, and so on (provided that we can neglect air resistance). This was one of Galileo's great discoveries, nearly 400 years ago. Figure 2.4 shows that Galileo was right.


Figure 2.4. Repeating the same experiment with a lighter or heavier falling object yields the same results, so long as air resistance is negligible. The golf ball (left) and bowling ball (right) are traveling at the same speed as each other at every point on the way down, despite their very different masses.

But our results are even more general than that. Suppose that instead of dropping the ball straight down, we attach it to a cord and let it swing downward as a pendulum (see Figure 2.5). There are still no other forms of energy involved here besides gravitational and kinetic, so any gravitational energy lost must show up as kinetic energy gained. Therefore, after the ball has dropped a vertical distance of one meter, it should have a speed of $4.4 \mathrm{~m} / \mathrm{s}$. (This time its direction of motion will no longer be straight down, but that doesn't matter because direction doesn't enter the formula for kinetic energy.)

Or consider a completely different situation: a roller-coaster on a frictionless track. If it starts out momentarily at rest, then rolls downhill a vertical distance of one meter, its speed at that point will again be $4.4 \mathrm{~m} / \mathrm{s}$, because all of the


Figure 2.5. These composite images were made in the same way as those for freely falling objects above, with successive images separated in time by $1 / 20$ second. At left is a low-friction cart with a black "flag" rolling down an inclined track; at right is a billiard ball attached to a string to make a pendulum. The horizontal lines on the wall are separated by half a meter, while the marks on the paper rulers are separated by 10 cm . In each case, after falling a vertical distance of one meter, the object's speed is approximately $4.4 \mathrm{~m} / \mathrm{s}$.
gravitational energy lost gets converted to kinetic energy.
Exercise 2.11. Calculate the kinetic energy of a $1500-\mathrm{kg}$ car moving at a speed of 65 mph . (Be sure to convert the speed to $\mathrm{m} / \mathrm{s}$ before plugging into the kinetic energy formula.)
Exercise 2.12. Consider again the example of dropping a ball whose mass is 0.2 kg . Calculate the gravitational energy lost by the ball upon falling three meters. Then estimate the speed at the three-meter point from Figure 2.3, and from the speed, calculate the kinetic energy gained. Do your results agree, within the range of uncertainty of your speed estimate?

Exercise 2.13. You now know how to predict the final speed of an object dropped from any height. But suppose, instead, that you wish to predict how long it takes an object to fall a certain distance - say two meters-when dropped from rest. As shown above, the final speed of this object after the two-meter fall is $6.3 \mathrm{~m} / \mathrm{s}$.
(a) How long would it take to make the descent, if it were moving this fast the whole way down?
(b) Explain why the actual time to make the descent must be longer than your answer to part (a).
(c) It's reasonable to guess that the average speed of a dropped object is half its final speed. And in fact, this guess turns out to be correct. Using the average speed instead of the final speed in your calculation, find the time needed for a dropped object to fall a distance of two meters. Check your answer using the data in Figure 2.3.

Exercise 2.14. Use conservation of energy to predict the speed of a dropped object that has fallen a distance of half a meter. Then check that each of the objects in Figures 2.3, 2.4, and 2.5 has approximately the predicted speed at the half-meter point.

Exercise 2.15. A roller-coaster is essentially motionless at the top of a hill. It then coasts downward, falling a vertical distance of 20 meters. How fast is it going at the bottom of the hill? (Hint: The roller-coaster's mass will cancel out of the calculation, so feel free to make up any value for the mass if you like.)

Exercise 2.16. Imagine throwing a stone horizontally, at a speed of $20 \mathrm{~m} / \mathrm{s}$, from the top of a 100-meter cliff. How fast will the stone be going when it hits the ground below the cliff?

Exercise 2.17. Imagine throwing a stone straight up into the air at a speed of $25 \mathrm{~m} / \mathrm{s}$. How high will it go before it runs out of kinetic energy?

Exercise 2.18. In a wind turbine (like a "windmill" but with an electric generator), the kinetic energy of the air is partially converted into electrical energy. Consider a wind turbine with a radius of 80 feet, in a location (Wyoming?) where the density of air is $1.0 \mathrm{~kg} / \mathrm{m}^{3}$ and the wind speed is 30 miles per hour. In one hour, the air that passes "through" the wind turbine can be visualized as a long cylinder, 30 miles long with a radius of 80 feet. Calculate the total kinetic energy of this air, first in joules and then in kilowatt-hours. Be sure to start by converting all numbers to official scientific units. (Comment: The efficiency of a wind turbine at extracting this energy is usually fairly low, perhaps $20 \%$ on average.)
Exercise 2.19. Paddy the bricklayer (whose mass is 75 kg ) has 100 kg of leftover bricks that he needs to bring down from the top of a 14 -story building. To avoid hauling them down by hand, he hangs a rope from a pulley and hoists a barrel to the top, tying off the rope at ground level, 50 meters below the barrel. After loading the bricks into the barrel, he goes back down to untie the rope, clinging tightly to it. Much to his surprise (as he has not studied physics), he begins to accelerate upwards. How fast are Paddy and the barrel moving when they collide into each other, 7 floors ( 25 meters) above the ground? (Hint: First calculate how much gravitational energy is converted into kinetic energy as the barrel falls and Paddy rises.)

## Work: The Transfer of Energy

Consider again the example of lifting a brick, giving it gravitational energy. The amount of energy you give it is given by its weight times the vertical distance lifted, and its weight is also equal to the force that you must apply in order to balance gravity as you hold it. Thus, the energy that you give it can be expressed as

$$
\begin{equation*}
\text { Energy transferred }=(\text { force applied }) \times(\text { distance moved }) \tag{2.15}
\end{equation*}
$$

On the other hand, if you carry the brick in a horizontal direction, although you still need to exert an upward force to support it, you are not transferring any energy to the brick. Apparently, motion along the direction of the force (here upward) transfers energy to an object, but motion perpendicular to the applied force does not transfer energy.

It turns out that this result is completely general. Whenever you exert a force on an object, moving the object in the direction of the applied force, you transfer energy to it. In the example of the brick, that energy showed up as gravitational energy, but in other cases, the energy could show up as kinetic energy or in or
some other form. Any such transfer of energy from one object to another is called mechanical work. To calculate the amount of mechanical work, all you have to do is multiply the force exerted by the distance traveled in the direction of the force:

$$
\begin{equation*}
\text { Mechanical work }=(\text { force }) \times(\text { distance parallel to force }) . \tag{2.16}
\end{equation*}
$$

Please notice that this use of the word "work" is a very specific and technical one, only distantly related to the meaning of the word in everyday English. For instance, while you might consider it hard work to walk a mile on a level road while wearing a 60 -pound backpack, technically you're doing no work at all because neither you nor the backpack is gaining any mechanical energy.

As an application of the idea of mechanical work, suppose that while throwing a baseball horizontally, you exert a force of 20 newtons over a distance of 1.5 meters. The amount of energy you've then transferred to the ball is

$$
\begin{equation*}
\text { work }=(20 \mathrm{~N})(1.5 \mathrm{~m})=30 \mathrm{~N} \cdot \mathrm{~m}=30 \mathrm{~J}, \tag{2.17}
\end{equation*}
$$

and so its kinetic energy as it leaves your hand must be 30 joules. Applying equation 2.10 in reverse, we can then conclude that the baseball's speed, as it leaves your hand, is $20 \mathrm{~m} / \mathrm{s}$.

Exercise 2.20. How much work must you perform to lift a $5-\mathrm{kg}$ bag of groceries from the floor to the countertop, one meter above the floor? (Hint: First calculate the weight of the bag of groceries, in newtons.)

Exercise 2.21. How much work must you perform to carry a $5-\mathrm{kg}$ bag of groceries from the car to the kitchen, if the total distance traveled is 15 meters and the total elevation gained is one meter?

Exercise 2.22. During a golfer's swing, the head of the club might be in contact with the ball over a distance of 0.6 m . If the ball's mass is 46 g and its maximum speed as it leaves the club is 150 mph , estimate the average force exerted on the ball by the club. (Hint: First convert all numbers to scientific units. Then calculate the ball's kinetic energy as it leaves the club.)

Exercise 2.23. A mover needs to lift a 250 -kilogram piano from the sidewalk up to the truck bed, a vertical distance of 1.2 meters.
(a) How much work would be required to lift the piano if the mover lifts it vertically?
(b) Instead of trying to lift the piano vertically, the mover wisely chooses to push it up an inclined ramp that is six meters long (with the same vertical ascent). Argue that the amount of work required to push the piano up the ramp should be the same as what is required to lift it vertically (neglecting friction in the piano's wheels).
(c) Calculate the force required to push the piano up the ramp, and compare to the force that would be required to lift it vertically.

Exercise 2.24. A weight lifter bench-presses a barbell whose mass is 150 kg , lifting it a distance of $1 / 3$ meter during each repetition.
(a) What is the weight of the barbell?
(b) How much work does the weight lifter perform during the upward motion of lifting the barbell once?
(c) Argue that when the weight lifter lowers the barbell, the work done must be a negative number: minus the amount needed to lift the weight. (Hint: What happens to the barbell's gravitational energy as it comes down?)
(d) After completing twenty repetitions, how much work has the weight lifter performed?

## Elastic Energy

When you stretch or compress something, you store elastic energy in it. How much energy depends on how far you stretch or compress it, and also on how "stiff" the object is. For instance, it takes a lot more energy to stretch a garage door spring than a small rubber band.

As long as you don't stretch an object too far, the amount of force that you must exert (or that it exerts on you) turns out to be directly proportional to the amount of stretch or compression. For instance, if it takes one newton of force to stretch a spring by ten centimeters, then two newtons of force are required to stretch it by twenty centimeters, and ten newtons will stretch it by a full meter. We can write this rule as the following equation:

$$
\begin{equation*}
\text { Spring force }=(\text { spring constant }) \times(\text { amount of stretch or compression }) \tag{2.18}
\end{equation*}
$$

or in symbols,

$$
\begin{equation*}
F_{\text {spring }}=k_{s} \cdot x \tag{2.19}
\end{equation*}
$$

where $x$ stands for the amount of stretch or compression and $k_{s}$, the spring constant, is a measure of the stiffness of your particular spring (or rubber band, etc.). More precisely, the spring constant is the amount of force required, per unit stretch or compression. The spring mentioned above has a spring constant of ten newtons per meter. A typical slinky has a spring constant of $0.5 \mathrm{~N} / \mathrm{m}$, while a heavy garage-door spring might have a spring constant of $500 \mathrm{~N} / \mathrm{m}$.

To calculate the energy stored in a spring, we can apply our formula for mechanical work to the process of stretching the spring:

$$
\begin{equation*}
\text { Energy transferred }=(\text { force applied }) \times(\text { distance stretched }) \tag{2.20}
\end{equation*}
$$

There's a subtlety, however, in determining what force to plug into this formula. During the process of stretching the spring, the force that you must exert grows from zero (at the very beginning) to a final value of $k_{s} \cdot x$, if $x$ is the final amount of stretch. The average force is therefore only half the final value, $\frac{1}{2} k_{s} \cdot x$, and this average is what we should plug into the energy formula:

$$
\begin{align*}
\text { Energy transferred } & =(\text { average force }) \times(\text { distance stretched }) \\
& =\left(\frac{1}{2} k_{s} \cdot x\right) \cdot x  \tag{2.21}\\
& =\frac{1}{2} k_{s} \cdot x^{2}
\end{align*}
$$

The amount of energy stored in the spring is therefore given by the same formula,

$$
\begin{equation*}
E_{s}=\frac{1}{2} k_{s} \cdot x^{2}, \tag{2.22}
\end{equation*}
$$

or in words,

$$
\begin{equation*}
\text { Spring energy }=\frac{1}{2}(\text { spring constant }) \times(\text { amount of stretch or compression })^{2} . \tag{2.23}
\end{equation*}
$$

This formula applies to any elastic object, whether stretched or compressed, as long as the stretch or compression isn't too extreme.

For example, consider again a medium-sized spring with a spring constant of $10 \mathrm{~N} / \mathrm{m}$. Stretching this spring a distance of half a meter would store

$$
\begin{equation*}
E_{s}=\frac{1}{2}(10 \mathrm{~N} / \mathrm{m})(0.5 \mathrm{~m})^{2}=1.25 \mathrm{~N} \cdot \mathrm{~m}=1.25 \mathrm{~J} \tag{2.24}
\end{equation*}
$$

of elastic energy in it. As the spring relaxes, this energy would be converted into some other form, for instance, kinetic or gravitational.

Exercise 2.25. A large garage-door spring has a spring constant of $500 \mathrm{~N} / \mathrm{m}$. How much force must you exert to stretch this spring by 5 cm ?

Exercise 2.26. A mass of 0.5 kg hangs motionless from a spring whose spring constant is $10 \mathrm{~N} / \mathrm{m}$. How far does the spring stretch?
Exercise 2.27. I have a rubber band that stretches 10 cm when a 500 g mass hangs from it (motionless). What is its spring constant? How much would this rubber band stretch if I hang an additional 500 g from it, making 1 kg total?
Exercise 2.28. How much energy is stored in a garage-door spring, with $k_{s}=$ $500 \mathrm{~N} / \mathrm{m}$, when it is stretched a distance of 0.8 m ?
Exercise 2.29. As part of a college dormitory battle, you construct a catapult of surgical hose mounted on a window frame. A typical value of the "spring" constant for such a catapult might be $100 \mathrm{~N} / \mathrm{m}$.
(a) How much force is needed to stretch this catapult five meters from its relaxed position?
(b) How much elastic energy is stored in the catapult when it is stretched by five meters?
(c) A one-kilogram water balloon is placed in the catapult and launched toward a neighboring dormitory. Assuming that all of the elastic energy in the catapult is converted to kinetic energy of the water balloon, what is the balloon's speed as it leaves the catapult?

