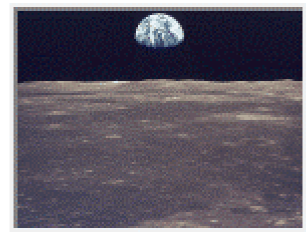
Name ______________________________ Section _______

Date _________



Measuring the Mass of the Earth

Objective

To derive the mass of the Earth using direct measurement of the acceleration of an object at the Earth's surface.

Introduction

Sir Isaac Newton changed the way in which humankind viewed the world. His laws describing the fundamental properties of physical reality took scientists from empirical work to mathematical logic. In particular, his description of gravity gave us a means to understand how we are bound to the Earth, how the moon is bound to the Earth, how the Earth is bound to the Sun, and so on. We now understand how one planet can perturb another or how a distant cluster of galaxies can "pull" us across immense distances.

Using Newton's formulae and knowledge of the radius of the Earth and the universal constant of gravitation, we can determine the mass of the Earth. We will be using the following equations:

1. $F = \frac{Gm_1m_2}{R^2}$	**2.** $F = m_2a$	**3.** $a = -2\frac{x}{t^2}$

where F is force, G (also known as **big G**) is the universal constant of gravitation (6.67×10^{-11} $m^3/kg\ s^2$), m_1 is the mass of the Earth, m_2 is the mass of an object on the surface of the Earth, a is the acceleration of that object (negative because the direction of the acceleration is down), x is the distance the object falls, and t is the amount of time it takes that object to fall that distance.

We can manipulate equations 1 and 2 to get the mass of the Earth in terms of the acceleration, radius, and constant of gravitation (your instructor will show you how):

4. $$m_1 = \frac{aR^2}{G}$$

Since we "know" R to be 6,378 km (=6,378,000 m; let's say we measured it somehow by observing the length and angles of shadows at different places on the Earth's surface) and G was fortunately measured for us by a physicist, all we need is a, the acceleration of an object at the Earth's surface.

We don't need to know the mass of the object. (Why is this true?) Although any object can be dropped, use an object that will experience a minimum of air resistance. Drop the object from a height that is high enough to minimize the relative fraction of time it takes us to start and stop a stop watch, but not so high that air resistance starts to affect the results.

Procedure

Find a location suitable for dropping stones or marbles. Examples of locations include open stairwells, balconies, and rooftops. Measure the distance the objects will be falling. Use a stopwatch with a precision of 1/100 of a second, and time how long each object takes to fall this distance. Work in groups and determine who will be dropping, who will be timing, who will be recording, who will measure the height, who has the calculator, who will be quality control, etc.

Exercise

Record the times in the following chart:

Trial No.	Time (sec)	Trial No.	Time (sec)	Trial No.	Time (sec)	Trial No.	Time (sec)
1		**6**		**11**		**16**	
2		**7**		**12**		**17**	
3		**8**		**13**		**18**	
4		**9**		**14**		**19**	
5		**10**		**15**		**20**	

Distance the object(s) fell (*x*): ________ meters Uncertainty: ________ meters

Share this data with the other members of your team. For mathematical understanding, each team member should do the following calculations individually. Again, please show all calculations here.

1. Calculate the average time taken to fall the distance, *x*:

   Average time: ___________________ seconds

2. Calculate the uncertainty in this average (via an ***approximate*** method):
   - Toss out the longest and shortest times
   - Subtract the now-shortest time from the now-longest time and divide by 2

   Uncertainty: _________ seconds

3. Square the value of the average time of fall:

   t^2 = ______________ sec^2

4. Solve for the acceleration of the object:

   $a = -2x/t^2$ = __________ m/s^2

5. Since the negative sign for the acceleration refers to direction, we can disregard it when determining the mass of the Earth. Solve for the mass of the Earth:

$$m_1 = aR^2/G = \_\_\_\_\_\_\_\_\_\_\_\_ \; kg$$

Questions

1. Why is it a good idea to take many measurements and average the results?

2. Compare your value for the mass of the Earth to the true value of 6×10^{24} kg. That is, calculate the percentage difference:

3. How does your value for the acceleration compare to the actual value of 9.8 m/s^2? With this answer in mind, how does your measurement for the acceleration of an object affect your derived value for the mass of the Earth?

4. Given your estimates for the uncertainties in the distance and times, what is the range of values allowed for the acceleration due to gravity at the surface of the Earth? That is, what are the lowest and highest values you find given the uncertainties of your experiment?

   - Highest value comes from combining the highest possible height with the shortest possible time, given your uncertainties. Calculate the highest value for the acceleration:

   - Lowest possible value comes from combining the shortest possible height with the longest possible time, given your uncertainties. Calculate the lowest value for the acceleration.

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5. Does the real value for the mass of the Earth lie within your uncertainties? (You can figure this out without doing any additional calculations.) Explain.