Force Vectors and Static Equilibrium

Overview:
In this experiment you will hang weights from pulleys over the edge of a small round “force table”, to exert various forces on a metal ring in the center of the table. You will check whether forces combine as vectors, and whether the vector sum of the forces is truly zero when the object is not accelerating.

Physics principles:
- Vector addition
- Vector components Force
- The gravitational force
- Newton’s second law

New lab skills:
- Describing the uncertainty of a measurement whose expected result is zero

Equipment needed:
- Force table
- 4 Pulleys
- 4 Weight holders
- Set of Weights
- Small metric ruler
- Protractor

Forces and Equilibrium
The “force table” that you will use in this experiment is shown in Figure 1 on the following page. In the center of the table is a small metal ring, which is pulled on by strings. You could just pull on the strings with spring scales that measure the forces exerted, but instead, for convenience, you will run the strings over pulleys and hang weights off their ends. As long as the weights are not accelerating and the pulleys are frictionless, the tension force exerted by each string then has the same magnitude as the gravitational force exerted by the earth on the weight:

\[ F_{\text{tension}} = F_{\text{gravity}} = M g. \]  

Here \( M \) is total mass of the hanging weight, including the weight holder whose mass is 50 g. The constant \( g \) is the strength of the local gravitational field, which near earth’s surface has the value

\[ g = 9.8 \, \text{N/kg} = .0098 \, \text{N/g}. \]
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Figure 1: Four forces in balance on the force table.

(You may be used to thinking of \( g \) as the acceleration of a freely falling object, which it also is. However, in this experiment nothing is falling. Since a newton (N) is a kg-m/s\(^2\), a newton per kilogram is the same as a meter per second squared.)

So the strings, weights, and pulleys give you a convenient way to exert various forces on the little metal ring. According to Newton’s second law, these forces determine the acceleration of the ring:

\[
\vec{a} = \frac{\sum \vec{F}}{m}.
\]

That is, the acceleration of the ring equals the vector sum of all the forces acting on it, divided by its mass \( m \). The idea in this experiment, though, is to adjust the forces so that the ring does not accelerate. Then Newton’s second law predicts

\[
\sum \vec{F} = 0 \quad \text{(when } \vec{a} = 0),
\]

and we say that the object is in “static equilibrium”.

In describing the various forces acting on the ring, we will need to adopt a coordinate system (see Figure 2). The center of the force table will define the origin of the \( xy \) plane, with the positive \( x \) axis pointing outward toward the point marked 0 on the rim. All angles will be measured counter-clockwise from this direction. Some of the force tables have two sets of numbers around the perimeter, one set increasing counter-clockwise and the other set increasing clockwise; use the set that increases counter-clockwise.

**Instructions**

1. **Balancing the Force Table**

Your first task is to set up the force table with four balancing forces that we will call \( \vec{A} \), \( \vec{B} \), \( \vec{C} \), and \( \vec{D} \). The first three of these forces will be at the angles 30\(^\circ\), 70\(^\circ\), and 180\(^\circ\),
respectively, so place three pulleys at these angles. The masses hanging from these three pulleys should be 150 g, 200 g, and 250 g, respectively, including the 50 g mass of the weight holder. Set up these three forces, using the pin in the middle of the table to hold the ring in place. Then remove the pin, while holding on to the fourth string to keep the ring centered. Note the angle at which you’re holding the fourth string. Put the fourth pulley there, and hang weights from it until the force table is in balance. The force exerted by this fourth string is $\vec{D}$.

Turn to the Report and fill in all the empty spaces of Table 1. Notice that the masses in the second column include the 50 g mass of the weight holder. For the third column compute the magnitude of each force in newtons. For the last two columns compute the $x$ and $y$ components of each force in newtons. Note that some of these components will be negative. Refer to your textbook if you don’t remember how to compute vector components. As a check, you should get approximately $-2.6$ N for the $y$ component of $\vec{D}$.

2. Estimating Uncertainties

Your next task is to estimate the uncertainty due to random error in your experimental determination of $\vec{D}$, the force needed to balance the other three. First try adding and subtracting a few grams on the weight holder for force $\vec{D}$, to obtain a range of values of this mass for which the ring is still centered. Write down this range in the Report. Then try adjusting the position of the pulley, to obtain a range of uncertainty for the angle, and write down this range. Finally, disassemble all the weights, rotate the ring so that a different string goes over each pulley, and put everything back together as you did originally, to obtain a completely new measurement of $\vec{D}$. If your new measurement lies outside the ranges already determined, adjust them accordingly.

A good measure of the uncertainty in a value is half the difference between the maximum possible value and the minimum possible value:

$$\text{uncertainty} = \frac{\text{maximum} - \text{minimum}}{2}.$$  \hspace{1cm} (5)
3. Testing Vector Addition of Forces

Now you are ready to test the idea that forces combine as vectors. Consider forces $\vec{A}$ and $\vec{B}$. What single force would have the same effect as the combination of these two? Answer this question experimentally by dismantling the weights and pulleys for $\vec{A}$ and $\vec{B}$, then setting up a new force in such a way as to rebalance the ring. This force is called the resultant of $\vec{A}$ and $\vec{B}$, or $\vec{R}_{AB}$. Enter the magnitude and angle of $\vec{R}_{AB}$ in the Report.

According to Newton, you can calculate the resultant $\vec{R}_{AB}$ by computing the vector sum of $\vec{A}$ and $\vec{B}$. (This principle is sometimes called “Newton’s fourth law”, because in his book he stated it immediately after the other three.) To test this principle, carefully draw the vectors $\vec{A}$ and $\vec{B}$ on the graph paper in the Report, using a scale of 4 cm per newton. Put the tail of $\vec{B}$ at the tip of $\vec{A}$, then connect the free ends to draw the vector sum $\vec{A} + \vec{B}$. Measure the magnitude and angle of this vector with a ruler and protractor, and enter these values in the Report.

A slightly more accurate method of computing $\vec{A} + \vec{B}$ is to use the laws of sines and cosines (see Figure 3). Recall from trigonometry that for any triangle with sides $a$, $b$, $c$, and opposite angles $\alpha$, $\beta$, $\gamma$, the following relations hold:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad \text{(law of cosines)}; \quad (6)$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \quad \text{(law of sines).} \quad (7)$$

Use these relations to compute the magnitude and angle of $\vec{A} + \vec{B}$, and enter the results (and your work) in the Report. (Your results should be very close to the values you measured graphically.)

A third method of adding vectors is to separately add their $x$ and $y$ components to obtain the components of the vector sum. Knowing the components of the sum, you can then use the Pythagorean theorem and a tangent function to obtain the magnitude and angle. Do this arithmetic and enter the results in the Report. (These results should agree exactly with those obtained using the laws of sines and cosines.)

![Figure 3: Side and angle definitions for applying the laws of sines and cosines.](image-url)
Before going on, answer Question 2.

Next, remove the weights and pulley for force $\vec{R}_{AB}$, replace the weights and pulley for force $\vec{A}$, remove the weights and pulley for force $\vec{C}$, and experimentally determine the force $\vec{R}_{BC}$ whose effect is the same as that of $\vec{B}$ and $\vec{C}$ combined. Enter its magnitude and angle in the Report, then compute the vector sum $\vec{B} + \vec{C}$ in three ways (graphically, using the laws of sines and cosines, and using components) just as before. Enter all results in the Report and answer Question 3.

4. The Law of Static Equilibrium

Now let us return to Newton’s second law, for the special case of zero acceleration:

$$\sum \vec{F} = 0 \quad \text{(when } \vec{a} = 0)\quad (8)$$

This law predicts that the sum of your original four forces, $\vec{A} + \vec{B} + \vec{C} + \vec{D}$, should be zero. To test whether the sum is zero, draw all four vectors tip-to-tail on the next piece of graph paper in the Report, then use a ruler to measure the magnitude of the sum of the four vectors. As a check, separately add the $x$ components and $y$ components of the four vectors, enter these values, and use the Pythagorean theorem to compute the magnitude of the sum. Answer Questions 4 and 5.
Report: Force Vectors and Static Equilibrium

Name ___________________________
Partners _________________________
Lab Station _______________________
Date _____________________________

1. Balancing the Force Table

Table 1

<table>
<thead>
<tr>
<th></th>
<th>mass (g)</th>
<th>magnitude (N)</th>
<th>angle</th>
<th>x component (N)</th>
<th>y component (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{A}$</td>
<td>150</td>
<td></td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{B}$</td>
<td>200</td>
<td></td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{C}$</td>
<td>250</td>
<td></td>
<td>180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vec{D}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Estimating Uncertainties

Range of masses for $\vec{D} =$ _____________________________

Range of angles for $\vec{D} =$ _____________________________

Uncertainty in magnitude of force $\vec{D} =$ ______________

Uncertainty in angle of force $\vec{D} =$ ______________
Question 1: What features of the equipment cause there to be uncertainties in the balancing force? List as many as you can, and for each, explain in a sentence how it would affect the balancing force.

3. Testing Vector Addition of Forces

Measured magnitude of $\vec{R}_{AB} =$ ___________  angle = ___________

Graphical construction of $\vec{A} + \vec{B}$:
From graph: Magnitude of $\vec{A} + \vec{B} =$ ____________  
angle = ____________

Computations using laws of sines and cosines:

Magnitude of $\vec{A} + \vec{B} =$ ____________  
angle = ____________

Component computations (from data in Table 1):

$x$ component of $\vec{A} + \vec{B} =$ ____________  
$y$ component = ____________

Computation of magnitude and direction from components:

Magnitude of $\vec{A} + \vec{B} =$ ____________  
angle = ____________

Question 2: Taking experimental uncertainties into account, are your computations of the magnitude and angle of $\vec{A} + \vec{B}$ in agreement with your measured magnitude and angle of $\vec{R}_{AB}$? Explain briefly.
Measured magnitude of $\vec{R}_{BC} =$ \underline{} \hspace{1cm} angle = \underline{}

Graphical construction of $\vec{B} + \vec{C}$:

From graph: Magnitude of $\vec{B} + \vec{C} =$ \underline{} \hspace{1cm} angle = \underline{}

Computations using laws of sines and cosines:

Magnitude of $\vec{B} + \vec{C} =$ \underline{} \hspace{1cm} angle = \underline{}

$x$ component of $\vec{B} + \vec{C}$ (from Table 1) = \underline{} \hspace{1cm} $y$ compon. = \underline{}

Computations from components:
Question 3: Taking experimental uncertainties into account, are your computations of the magnitude and angle of $\vec{B} + \vec{C}$ in agreement with your measured magnitude and angle of $\vec{R}_{BC}$? Explain briefly.

4. The Law of Static Equilibrium

Graphical construction of $\vec{A} + \vec{B} + \vec{C} + \vec{D}$:

Measured magnitude of $\vec{A} + \vec{B} + \vec{C} + \vec{D} =$ _____________
Sum of $x$ components (from Table 1) = 

Sum of $y$ components (from Table 1) = 

Magnitude of $\vec{A} + \vec{B} + \vec{C} + \vec{D}$ (from components) = 

Question 4: Are your determinations of the magnitude of $\vec{A} + \vec{B} + \vec{C} + \vec{D}$ consistent with the predicted value of zero? Explain.

Question 5: Often when we compare a measured quantity with a theoretical prediction, we compute the percent difference between the two as follows:

$$\text{percent difference} = \frac{(\text{measured}) - (\text{theoretical})}{(\text{theoretical})} \times 100.$$ 

Explain why this would not be a meaningful computation for the magnitude of the vector sum $\vec{A} + \vec{B} + \vec{C} + \vec{D}$. Can you suggest a remedy? That is, for what quantity in this experiment would the percent difference be meaningful?