The Simple Pendulum

Overview:
In this lab you will determine how the period of a simple pendulum (a billiard ball hanging from a string) depends on its length and on its amplitude of oscillation. You will also use the pendulum to make an accurate measurement of the gravitational constant \( g \).

Physics principles:
Oscillatory motion of a pendulum

New lab skills:
Using “error bars” on a graph
Reading a vernier caliper
Testing the range of validity of an approximate formula

Equipment needed:
Ball on a long string
Support for the pendulum
Vernier caliper
Two-meter stick
Protractor
Computer and ScienceWorkshop interface
Photogate

Instructions
You'll use a photogate to measure the period of the pendulum. Connect the photogate into port 1 of the interface box. Open the “Simple Pendulum” document in the “Lab Files” folder of the computer's desktop. A data table is set up to measure the time it takes for the pendulum to complete one full oscillation (one period), along with the Mean (average period) and the standard deviation. Click “Setup” and “Setup Timers” to see the Timing Sequence for “Timer 1.” With your lab partners, determine why this sequence works. Deselect “Setup” to hide the setup window and proceed with the activity.

1. Dependence of Period on Amplitude
In this first part of the experiment you will study how the period of the pendulum depends on the amplitude, or maximum angle, of the swing.

Clamp the string to the support so that the length of the pendulum is somewhere between 50 and 70 cm. Attach a protractor to the pendulum support bar in such a way...
that it does not interfere with the swing of the pendulum, but still allows you to measure the amplitude of swing. **Note:** The pendulum support system must be braced so that it will not move as the pendulum swings back and forth. Check this before you take data, by pulling the ball back some distance and letting it go. The support rod must not sway along with it. If necessary, have your instructor help you secure your support system.

Place the photogate at the bottom of the pendulum swing so that the brass tube on the bottom of the ball passes through the gate as the pendulum swings back and forth. Pull back the pendulum ball so that the angle of swing (i.e., the maximum angle from the vertical) is 5°. Release the ball and be sure it passes freely through the photogate. Click the “Start” button on the screen to begin taking data as the pendulum continues to swing. The computer should record several successive periods in the “Elapsed Time” column, and the Mean and Standard Deviation should appear at the bottom of the table. Click the Stop button after five or more values have been recorded. If the pendulum gets out of line with the photogate, stop it and start it over. Record the information called for in Table 1, keeping digits down to the third decimal place (that is, to the nearest thousandth of a second).

Repeat this timing procedure for angles of swing of 10, 15, 20, 30, 40, 50, and 60 degrees. Restart the data collection with each new angle by clicking “Start”. It may be helpful to rename each data run by its angle measurement (i.e., “Run 1” would be changed to “Run 5”). If you want to do this, click the Run you want to change (under the Timer 1 (s) heading in the Data table), wait a second, then click it again and type the new name in the box. Note that for large amplitudes you may notice a significant decay in the amplitude over a few oscillations; it is therefore best not to continue timing beyond about five cycles.

**Analyzing the Data**

Launch the Excel program and create a spreadsheet of the data in Table 1, putting the angles in the first column and the periods in the second column. (You need not enter the
standard deviations.) Make a scatter plot of period vs. angle, giving it the title “Figure 1 — Period vs. Angle”. Enlarge the graph to fill a page, then print copies for each person in your group.

Your graph shows the average period for each of your runs, but not the standard deviation (which gives a rough measure of the uncertainty). To include information about uncertainties on a graph, it is standard scientific practice to draw an “error bar”, or vertical line, through each point, to indicate the range of uncertainty. Add error bars to your graph by hand, drawing a line both upward and downward by one standard deviation from each point. (Since the standard deviation is only a rough measure of the uncertainty, you needn’t be too meticulous with the lengths of the error bars.) Answer Question 1.

Now use a pencil to draw a smooth curve through your data points. Try to draw the smoothest curve you can that passes through, or at least close to, every error bar. Answer Questions 2 and 3.

**Some Theoretical Background**

The standard formula for the period of a pendulum is

\[ T = 2\pi \sqrt{\frac{L}{g}}, \]  

(1)

where \( L \) is the length of the pendulum and \( g \) is the gravitational constant. This formula has a number of important features, but notice first that it is independent of the amplitude of oscillation. Your data should show that the amplitude actually does affect the period, though only by a few percent. What’s going on here?

In fact, the standard formula is only an approximation, valid in the limit of very small angles. In deriving it one must pretend that the gravitational torque exerted on the pendulum is directly proportional to its angle from the vertical. In fact, the torque is proportional to the sine of the angle. In the limit of small angles, \( \sin \theta \approx \theta \) (in radians) so the proportionality holds and the formula for \( T \) is valid. But for large angles, deviations from the standard formula become noticeable.

From the approximately parabolic shape of your graph, you might guess that we could improve the standard formula by adding a term proportional to the square of the amplitude \( \theta_0 \). One way to write the improved formula would be

\[ T = 2\pi \sqrt{\frac{L}{g} \left( 1 + C\theta_0^2 \right)}, \]  

(2)

where \( C \) is some constant. With a lot of math one can show that the best theoretical value of \( C \) is exactly \( 1/16 \) rad\(^{-2}\), or about \( 1.9 \times 10^{-5} \) deg\(^{-2}\). If you have extra time, you might check whether your results agree with this prediction. However, even the improved formula (2) is not exact. One could improve it further by adding terms proportional to \( \theta_0^4 \) and so on, but let’s move on.

Going back to the basic formula (1), notice that the period is supposed to depend on both the length of the pendulum and the gravitational constant. In the next part of the experiment you will test the dependence on length, and extract from your data a measurement of \( g \).
2. Dependence of Period on Length

If all the mass of the pendulum were concentrated at a point, then the length $L$ that appears in formula (1) would be the distance of that point from the pivot. For your pendulum it is a good approximation to take $L$ to be the distance from the pivot down to the center of the billiard ball. (The best value of $L$ can be calculated from the pendulum’s moment of inertia, and turns out to be just a tiny bit more than the distance to the ball’s center. The tiny bit, though, is always less than 1% in this experiment, and is less than 0.1% when the pendulum is longer than 60 cm. So we’ll just take $L$ to be the distance to the center.)

To determine $L$ you need to know the diameter of the ball, which is hard to measure with a ruler. Instead, measure the diameter by placing the ball between the jaws of the vernier caliper. Examine the caliper carefully to see how it works, and have your instructor or lab aide show you how to read the scale. (You’ll be using the caliper and other vernier scales in other experiments, so make sure that everyone learns how to read it.) Enter the diameter and radius of the ball in the Report.

Clamp the string so that the length of the string is about 20 cm. Record the actual string length (including the hook at the top of the ball) in Table 2; you’ll add the ball’s radius to compute the pendulum length ($L$) later. Using the same procedure as in Part 1, measure the period of the pendulum. Keep the amplitude very small (less than 10°), so that formula (1) should be accurate to within a fraction of a percent. After timing for five or more swings, record the average period in Table 2. Repeat for lengths of 40, 60, 80, 100, 120, and 140 cm.

Set up a new Excel spreadsheet and enter the string lengths in column A. In column B, enter formulas to add the ball’s radius to obtain the pendulum length $L$. Enter your values of the period in column C. Put appropriate headings on all columns, and type your names in the first row.

Look again at the theoretical formula (1). By now you should know that because $T$ is supposed to be proportional to the square root of $L$, you will not get a straight line if you plot $T$ vs. $L$. Instead you should plot some simple function of $T$. Answer Question 4, then enter the appropriate function in column D of your spreadsheet. (If you are unsure of the correct function, be sure to check with your instructor or lab aide.)

Make a plot of column D vs. column B, to obtain the expected linear graph. Call this graph “Figure 2” with an appropriate descriptive title. Enlarge it to fill a page, then print copies of the data table and graph for each person. Carefully draw the best straight line through the points on your graph, and measure its slope. Answer Questions 5, 6, 7, 8 and 9.

Be sure to attach your computer printouts (Figures 1 and 2, and the data table used to plot Figure 2) to your Report.
Report: The Simple Pendulum

Name ____________________________
Partners __________________________
________________________
________________________
Lab Station _________________________
Date _____________________________

Table 1 — Dependence of Period on Amplitude

<table>
<thead>
<tr>
<th>Angle</th>
<th>5°</th>
<th>10°</th>
<th>15°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
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<tbody>
<tr>
<td># points</td>
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<td>std. dev. (s)</td>
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</tbody>
</table>

Question 1: Are your results consistent with the hypothesis that the period of a pendulum is independent of its amplitude of oscillation? Explain.

Question 2: According to the smooth curve you drew in Figure 1, what is the limiting value of the period as the amplitude goes to zero?

Question 3: According to the smooth curve you drew in Figure 1, how large must the amplitude be before the period is 1% larger than the limiting value you determined for the previous question? (Please show your work.)
Diameter of ball = ____________  Radius = ____________

Table 2 — Dependence of Period on Length

<table>
<thead>
<tr>
<th>Nominal L</th>
<th>20 cm</th>
<th>40 cm</th>
<th>60 cm</th>
<th>80 cm</th>
<th>100 cm</th>
<th>120 cm</th>
<th>140 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>String length (cm)</td>
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<td>mean $T$ (s)</td>
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</table>

Question 4: Based on equation 1, what function of $T$ should you plot vs. $L$ to obtain a straight-line graph? Please explain your reasoning.

Question 5: Explain why it would have been pointless to keep track of the standard deviations in this part of the experiment, and to use these to draw error bars on Figure 2.

Question 6: Does your Figure 2 verify the prediction that $T$ is proportional to the square root of $L$? Please explain.
Question 7: Derive a relation between the slope of your Figure 2 and the gravitational constant $g$, then calculate $g$ from your measured value of the slope.

Question 8: Make an estimate of the uncertainty in your value of $g$, and explain your reasoning. (Do not estimate the uncertainty by comparing to the “correct” value. Instead, pretend that you are the first person ever to measure this constant, and estimate the uncertainty based on the accuracy of your slope determination and other experimental factors. Express the uncertainty as a percentage.)

Question 9: The accepted value for $g$ in Ogden, based on U.S. Geological Survey measurements with precise pendulum stations, is 979.8 cm/s$^2$. Given your uncertainty estimate, does your measurement agree with this accepted value? Explain briefly.