AAPT Physics Education
The Physics Teacher

The Square Light Clock and Special Relativity

J. Ronald Galli and Farhang Amiri

Citation: The Physics Teacher **50**, 212 (2012); doi: 10.1119/1.3694069
View online: http://dx.doi.org/10.1119/1.3694069
View Table of Contents: http://scitation.aip.org/content/aapt/journal/tpt/50/4?ver=pdfcov
Published by the American Association of Physics Teachers

The Square Light Clock and Special Relativity

J. Ronald Galli and Farhang Amiri, Weber State University, Ogden, UT

A thought experiment that includes a square light clock is similar to the traditional vertical light beam and mirror clock, except it is made up of four mirrors placed at a 45° angle at each corner of a square of length L_0, shown in Fig. 1. Here we have shown the events as measured in the rest frame of the square light clock. By studying the same cycle of events measured in a frame of reference where the square light clock is in motion with speed of v, we can derive both time dilation and, independently, length contraction. This will be done by analyzing the changing dimensions and also the one-cycle timing for this square light clock.

The analysis is based on the two principles of Einstein for special relativity,[1] which may be stated as follows:

1. The physical laws are the same in all uniformly moving reference frames, independent of the relative motion of any particular frame.
2. The velocity of light in empty space is independent of the motion of the source or the motion of the observer.

In Fig. 1, in which the clock is at rest (the rest frame), the light beam is shown starting at position P_1, then reflecting from a mirror at P_2, then subsequently at P_3, then P_4, and finally returning to P_1. Figure 2(a) shows the same sequence of events in the frame of reference where the square light clock is set to a horizontal motion with speed v to the right. We shall call this frame the lab frame. Since the motion of the clock takes place in the horizontal direction, the vertical length L_0 remains unchanged in the lab frame. The invariance of transverse dimensions is discussed by other authors, for example, Refs. 2 and 3.

The important parameters are listed as follows:

a. L_0 is the length of each side of the clock in the rest frame.

b. L is the length of the longitudinal side of the moving clock in the lab frame.

c. T_0 is the total round trip time of light for the clock in the rest frame.

d. T is the total round trip time of light for the clock in the lab frame.

e. T_1 is the time from P_2 to P_3 in the lab frame.

f. T_2 is the time from P_4 to P_1 in the lab frame.

g. T_V is the total vertical travel time of light in the lab frame (P_1 to P_2 plus P_3 to P_4).

h. T_H is the total horizontal travel time of light in the lab frame ($T_1 + T_2$).

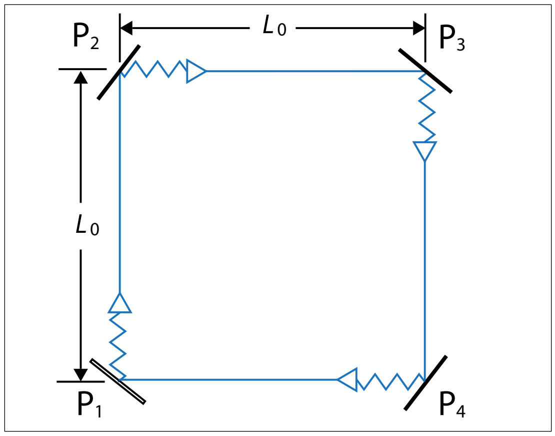


Fig. 1. Square light clock at rest showing beam as it starts at P_1, then reflects at P_2, P_3, P_4 and returns to P_1.

Therefore, based on the above definitions, we have $T_H = T_1 + T_2$. Further, the horizontal distance traveled by the clock as the beam of light travels from P_1 to P_2 is the same as when the beam travels from P_3 to P_4 [see Fig. 2(a)]. Therefore, from Fig. 2(a), it can be observed that $T_{1\rightarrow 2} = T_{3\rightarrow 4} = T_V/2$. Also note, as described below, that $T_V = T_H$, and consequently, $T_V = T_1 + T_2$.

The assertion that $T_V = T_H$ follows from the first principle of relativity that all clocks (digital, mechanical, optical, etc.) that have a common period in the rest frame must have a common period when viewed from the lab frame in which they are moving with a uniform velocity. Let us visualize two identical clocks at rest. Each clock consists of a source of light with a reflecting mirror at a given distance that returns the light to the source. One clock is oriented vertically as in the traditional derivation of the time dilation equation[2,3] and the other is oriented horizontally. For each clock, a light beam will be emitted and returned to its respective source in a common time interval. In a similar way, when these two clocks are observed from the lab frame in which they move uniformly together, the "round trip" time intervals will again be common, but different from that measured in their rest frame.

Next we compare the two-clock setup discussed above with the time measurements in the square light clock of Fig. 2(a), assuming the same speed for the lab frame. The period (i.e., the round trip time) of the clock placed horizontally, as measured in the lab frame, is equivalent to the time from P_2 to P_3, plus from P_4 to P_1 (although that happens later) in the square light clock. Similarly, the period (i.e., the round trip time) of the clock placed vertically, as measured in the lab frame, is equivalent to the time from P_1 to P_2, plus P_3 to P_4 (which also happens later) in the square light clock. Therefore,

 DOI: 10.1119/1.3694069

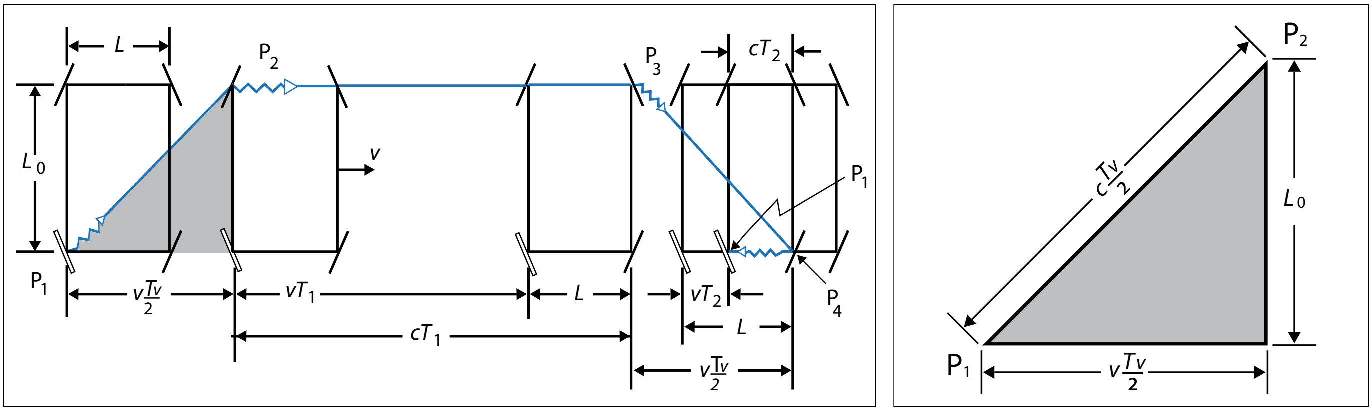


Fig. 2. (a) Square light clock of Fig. 1, but moving with speed *v*, shown with five sequential positions as the beam starts at P_1, reflects from each corner and returns to P_1. Here the scale is *v* /*c* ≈ 0.75.

Fig. 2. (b) First quarter-cycle of moving clock.

in the square light clock, the total vertical time T_V and the total horizontal time T_H would again be common and consequently $T_V = T_H$.

Length contraction

Consider the clock moving uniformly with speed v. As the light beam travels from P_2 to P_3, the geometry from Fig. 2(a) yields

$$L = cT_1 - vT_1. \qquad (1)$$

Similarly, as light travels from P_4 to P_5, the corresponding geometry yields

$$L = cT_2 + vT_2\,. \qquad (2)$$

Combining Eq. (1) and Eq. (2) yields

$$v(T_1 + T_2) = c(T_1 - T_2\,). \qquad (3)$$

And then using $T_V = T_1 + T_2$, we find

$$vT_v = c(T_1 - T_2\,). \qquad (4)$$

Now consider the path P_1 to P_2 or equivalently P_3 to P_4. From the highlighted triangle of Fig. 2(a) as shown in Fig. 2(b), we get

$$c^2(T_v/2)^2 = v^2\,(T_v/2)^2 + L_0^2\,,$$

or

$$c^2T_v^2 = v^2T_v{}^2 + 4L_0^2. \qquad (5)$$

Noting again that $T_v = T_1 + T_2$ and using Eq. (4), we may write Eq. (5) as

$$c^2(T_1 + T_2)^2 = c^2\,(T_1 - T_2\,)^2 + 4L_0^{\,2}. \qquad (6)$$

This may be reduced to

$$T_1T_2 = \frac{L_0^2}{c^2}. \qquad (7)$$

Solving Eq. (1) and Eq. (2) for T_1 and T_2 gives

$$T_1 = \frac{L}{c-v} \text{ and } T_2 = \frac{L}{c+v}.$$

Now Eq. (7) may be written in terms of L and L_0 as

$$\left(\frac{L}{c-v}\right)\left(\frac{L}{c+v}\right) = \frac{L_0^2}{c^2}. \qquad (8)$$

This finally reduces to

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}. \qquad (9)$$

This is the length contraction equation, which has been derived without explicit use of the time dilation equation.

Time dilation

Consider, again, the Fig. 2(b) triangle with $L_0 = \frac{cT_0}{4}$. We get

$$\frac{c^2T_V^2}{4} = \frac{v^2T_V^2}{4} + \frac{c^2T_0{}^2}{4^2}.$$

which simplifies to

$$c^2T_V^2 = v^2T_V^2 + \frac{c^2T_0{}^2}{4}. \qquad (10)$$

Then note again $T_V = T_H$ so that $T_V = \frac{T}{2}$ and therefore Eq. (10) may be written as

$$c^2\left(\frac{T}{2}\right)^2 = v^2\left(\frac{T}{2}\right)^2 + c^2\left(\frac{T_0}{2}\right)^2. \qquad (11)$$

This can be rearranged to

$$T^2 - \frac{v^2}{c^2}T^2 = T_0^2, \qquad (12)$$

which finally leads to

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad (13)$$

This is the time dilation equation, which was derived independent of the length contraction equation.

Discussion and conclusions

In this paper, we have demonstrated that both time dilation $T = T_0 / \sqrt{1 - v^2 / c^2}$ and length contraction $L = L_0 / \sqrt{1 - v^2 / c^2}$ can be derived independent from each other by employing a conceptually simple process. The traditional "vertical light clock" and "horizontally moving train" are combined into a single device where the horizontal length of the square light clock is measured independent of time dilation. It corroborates that length contraction does not depend explicitly on time dilation and vice versa, but only on the fundamental principles of the special theory of relativity.

The results, derived for a moving light clock, would apply to all time processes (clocks—electronic, mechanical, etc.) and all objects that are moving with the same uniform speed as the square light clock. This is due to the basic principle of special relativity, that within any given frame, all clocks that have a common period at rest will have a common period when observed in a uniformly moving frame. Also, all uniformly moving objects will maintain the same relative dimensions as if at rest. Thus, these results are general in their applicability to all clocks and all objects.

One interesting consequence of the square light clock geometry is the change of orientation of each of the mirrors. The length contraction causes the mirrors to assume a direction that makes their angle to be more than 45º. This change of orientation, plus the effects of the motion of the clock, causes the angle of incidence and the angle of reflection to not be equal as light is incident and reflected from each mirror.[4,5]

References

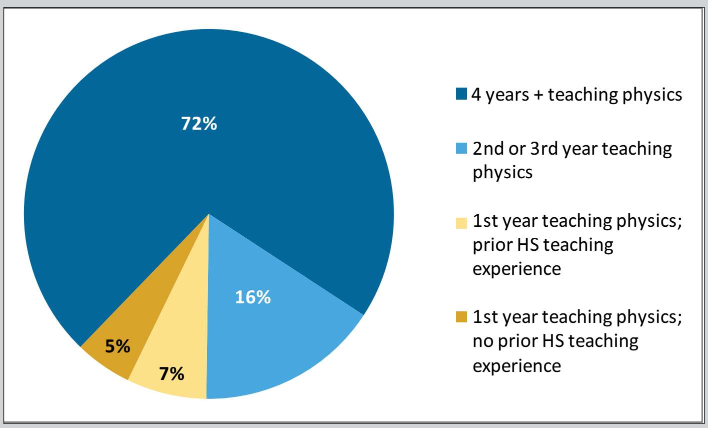
1. A. Einstein, "Zur Elektrodynamik Bewegter Korper," *Ann. Phys.* (Leipzig) **17**, 891–921 (1905), reprinted in Einstein's *Miraculous Years: Five Papers that Changed the Face of Physics*, edited by John Stachel (Princeton University Press, Princeton, 1998).
2. E. F. Taylor and J. A. Wheeler, *Space Time Physics*, 2nd ed. (W. H. Freeman and Company, New York, 1992), pp. 65–70.
3. R. Resnick, *Introduction to Special Relativity* (Wiley, New York, 1968), pp. 67-69.
4. A. Gjurchinovski, "Reflection of light from a uniformly moving mirror," *Am. J. Phys.* **72**, 1316–1324 (Oct. 2004).
5. B. M. Bolotoskii and S. N. Stolyyarov, "Reflection of light from a moving mirror and related problems," *Sov. Phys. Usp.* **32** (9), 813–827 (Sept. 1989).

J. Ronald Galli *is a professor of physics at Weber State University in Ogden, UT. He has served as chair of the physics department and then as dean of the College of Science. He is known for building a mechanical model to explain the cat twist. Along with Farhang Amiri, he has created and published videos of physics demonstrations.*
jrgalli@weber.edu; physics.weber.edu/galli

Farhang Amiri *is a professor of physics at Weber State University, with specialty in elementary particle physics. His interest is in developing materials used in teaching physics. His most recent projects include videos of physics demonstrations with Ronald Galli, and computer animations in physics with Brad Carroll.*
physics.weber.edu/amiri

High School Physics Teaching Experience

We divided our high school physics teaching experience into three groups: first year teaching physics, second or third year teaching physics, and four or more years of experience teaching physics. We did this because everything is new for teachers teaching a course for the first time. The second and third time through the course, teachers learn from past experiences and hone their approaches. By the time a teacher is in the fourth year of teaching a course, he or she is more comfortable with the material and better able to understand the ways in which different approaches work with different topics.



As shown in the figure, almost three-fourths (72%) of high school physics teachers have taught the course for four years or more, and about one high school physics teacher in six (16%) have taught physics for two or three years. Over half (~58%) of the "new" physics teachers—those teaching physics for the first time—have high school teaching experience in other subjects.

In the May issue, we will look at overall high school teaching experience among high school physics teachers. If you have any questions or comments, please contact Susan White at swhite@aip.org. Susan is Research Manager in the Statistical Research Center at the American Institute of Physics and directs the high school survey.

DOI: 10.1119/1.3694070