# Homework # 8

(32-1) a) At $\lambda_m = 0$,

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos\lambda_m$$

$$= \frac{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(8 \times 10^{22} A \cdot m^2)}{4\pi (6.37 \times 10^6 m)^3} \cos 0^\circ$$

$$= 3.095 \times 10^{-5} T \cos 0^\circ = 3.095 \times 10^{-5} T$$

and $B_V = \frac{\mu_0 \mu}{2\pi r^3} \sin\lambda_m$

$$= \frac{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(8 \times 10^{22} A \cdot m^2)}{2\pi (6.37 \times 10^6 m)^3} \sin 0^\circ$$

$$= 6.190 \times 10^{-5} T \sin 0^\circ = 0$$

So $\boxed{B = 3.095 \times 10^{-5} T}$

$$\phi_i = \tan^{-1} \frac{B_v}{B_h} = \tan^{-1}\left[\frac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m}\right] = \tan^{-1}(2\tan\lambda_m)$$

$$\Rightarrow \phi_i = \tan^{-1}(2\tan 0^\circ) = \boxed{0^\circ}$$

b) At $\lambda_m = 60^\circ$

$$B_h = 3.095 \times 10^{-5} T \cos 60^\circ = 1.55 \times 10^{-5} T$$

$$B_v = 6.190 \times 10^{-5} T \sin 60^\circ = 5.36 \times 10^{-5} T$$

$$B = \sqrt{B_h^2 + B_v^2} = \sqrt{(1.55 \times 10^{-5} T)^2 + (5.36 \times 10^{-5} T)^2}$$

$$= \boxed{5.58 \times 10^{-5} T}$$

$$\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(2\tan 60^\circ) = \boxed{73.90^\circ}$$

c) At $\lambda_m = 90°$,

$B_h = 3.095\times10^{-5}\,T \cos 90° = 0$

$B_v = 6.190\times10^{-5}\,T \sin 90° = 6.190\times10^{-5}\,T$

So $\boxed{B = 6.190\times10^{-5}\,T}$ twice the equatorial value!

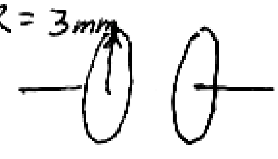
$\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(2\tan 90°) = \boxed{90°}$

(32-2) Iron ceases to be ferromagnetic at its Curie temperature, $T_c = 1043\,K = 770°C$. If the surface temperature is $10°C$ and the temperature increases by $30°C/km$, iron will cease to be ferromagnetic at a depth of

$$d = \frac{(770°C - 10°C)}{30°C/km} = \boxed{25.3\,km}$$

(32-3) First find the displacement current from

$$B = \frac{\mu_0 i_d}{2\pi r}$$




$$\Rightarrow i_d = \frac{2\pi r B}{\mu_0}$$

The displacement current is $i_d = \varepsilon_0 A \frac{dE}{dt}$
so

$$\varepsilon_0 A \frac{dE}{dt} = \frac{2\pi r B}{\mu_0}$$

$$\frac{dE}{dt} = \frac{2\pi r B}{\mu_0 \varepsilon_0 A} = \frac{2\pi r B}{\mu_0 \varepsilon_0 \pi R^2} = \frac{2 r B}{\mu_0 \varepsilon_0 R^2}$$

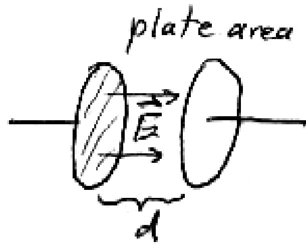
Thus

$$\frac{dE}{dt} = \frac{2(6\times10^{-3}m)(2\times10^{-7}T)}{(4\pi\times10^{-7}\frac{T\,m}{A})(8.85\times10^{-12}\frac{C^2}{Nm^2})(3\times10^{-3}m)^2}$$

$$= \boxed{2.40\times10^{13}\ \frac{N/C}{sec}}$$

(32-4) $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

$= \varepsilon_0 A \frac{dE}{dt}$




$\Phi_E = EA$

If the voltage between the plates is V, then

$$V = Ed \quad \text{or} \quad E = \frac{V}{d}$$

Then

$$i_d = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$$

$$i_d = \varepsilon_0 A \frac{1}{d}\frac{dV}{dt} \quad \text{(we seek } \frac{dV}{dt}\text{)}$$

But $\frac{\varepsilon_0 A}{d} = C$, the capacitance of a parallel plate capacitor. So

$$i_d = C\frac{dV}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{i_d}{C} = \frac{1.5\,A}{2\times10^{-6}F} = \boxed{7.5\times10^{5}\,\frac{V}{sec}}$$

is the rate at which the voltage must be changed

(32-5) a) $i_d = \varepsilon_0 A \frac{dE}{dt}$

$$= \varepsilon_0 A \frac{d}{dt}\left[(4\times10^5)\,\frac{N}{C} - (6\times10^4)\frac{N/C}{sec}\,t\right]$$

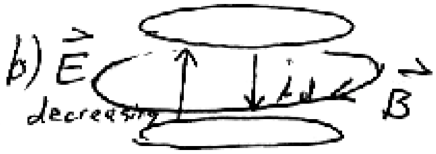
$$= -\varepsilon_0 A (6\times10^4)\frac{N/C}{sec}$$

$$= -(8.85\times10^{-12}\,\frac{C^2}{Nm^2})(4\times10^{-2}\,m^2)(6\times10^4\,\frac{N}{C\cdot sec})$$

$$= \boxed{-2.12\times10^{-8}\,A}$$

Since $\frac{dE}{dt} < 0$, (+) charge is running off the positive plate

$\Rightarrow i_d$ is in the opposite direction of $\vec{E}$

b) 


$\vec{B}$ is clockwise, seen from above the plates.

(33-6) a) $I = \frac{\mathcal{E}}{X_C} = \frac{\mathcal{E}}{1/\omega C} = \mathcal{E}\omega C$

$= (30\,V)(2\pi)(10^3 \frac{1}{sec})(1.5\times10^{-6}F)$

$= \boxed{0.283\,A}$

b) $I = \mathcal{E}\omega C = (30\,V)(2\pi)(8\times10^3\frac{1}{sec})(1.5\times10^{-6}F) = \boxed{2.26\,A}$

(33-7) a) $I = \frac{\mathcal{E}}{X_L} = \frac{\mathcal{E}}{\omega L} = \frac{30\,V}{2\pi(10^3\frac{1}{sec})(50\times10^{-3}H)} = \boxed{9.55\times10^{-2}A}$

b) $I = \frac{\mathcal{E}}{\omega L} = \frac{30\,V}{2\pi(8\times10^3\frac{1}{sec})(50\times10^{-3}H)} = \boxed{1.19\times10^{-2}A}$

(33-8) At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1H)(10^{-6}F)}} = 10^3\,\frac{rad}{sec}$

The impedence at resonance is

$$Z = \sqrt{R^2 + (\omega_0 L - \frac{1}{\omega_0 C})^2} = R \quad (\text{since } \omega_0 L = \frac{1}{\omega_0 C})$$

so

$$Z = 10\,\Omega$$

The current amplitude is, at resonance,

$$I = \frac{\mathcal{E}}{Z} = \frac{10\,V}{10\,\Omega} = 1\,A$$

The amplitude of the voltage across the inductor at resonance is

$$V_L = I X_L = I\omega_0 L = (1\,A)(10^3\,\tfrac{rad}{sec})(1\,H)$$

$$= \boxed{10^3\,V}\ !$$

(33-9) $\tan\phi = \frac{X_L - X_c}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$

$\Rightarrow \quad R = \frac{\omega L - \frac{1}{\omega C}}{\tan\phi}$

$= \frac{2\pi(930\frac{1}{sec})(88\times10^{-3}H) - \frac{1}{(2\pi)(930\frac{1}{sec})(0.94\times10^{-6}F)}}{\tan 75^\circ}$

$= \boxed{89\,\Omega}$

(33-10) a) $Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + X_L^2}$ since $X_c = 0$

$Z = \sqrt{(12\,\Omega)^2 + (1.3\,\Omega)^2} = 12.07\,\Omega$

b) $P_{ave} = \mathcal{E}_{rms} I_{rms} \cos\phi$

$= \mathcal{E}_{rms} \frac{\mathcal{E}_{rms}}{Z} \frac{R}{Z}$

$= \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = I_{rms}^2 R$

So $P_{ave} = \left(\frac{120\,V}{12.07\,\Omega}\right)^2 (12.07\,\Omega)$

$= \boxed{1.19\times10^3\ W}$

(33-11) $Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + X_L^2}$

$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{420\,V}{\sqrt{(32\,\Omega)^2 + (45\,\Omega)^2}} = \boxed{7.61\ A}$

(33-12) $V_s = V_p \frac{N_s}{N_p} = (100\,V)\left(\frac{500}{50}\right) = \boxed{1000\ V}$