

## Homework #8

(32-1) a) At  $\lambda_m = 0$ ,

$$B_h = \frac{\mu_0 \mu}{4\pi r^3} \cos \lambda_m$$

$$= \frac{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(8 \times 10^{22} A \cdot m^2)}{4\pi (6.37 \times 10^6 m)^3} \cos 0^\circ$$

$$= 3.095 \times 10^{-5} T \cos 0^\circ = 3.095 \times 10^{-5} T$$

$$\text{and } B_v = \frac{\mu_0 \mu}{2\pi r^3} \sin \lambda_m$$

$$= \frac{(4\pi \times 10^{-7} \frac{T \cdot m}{A})(8 \times 10^{22} A \cdot m^2)}{2\pi (6.37 \times 10^6 m)^3} \sin 0^\circ$$

$$= 6.190 \times 10^{-5} T \sin 0^\circ = 0$$

$$\text{So } \boxed{B = 3.095 \times 10^{-5} T}$$

$$\phi_i = \tan^{-1} \frac{B_v}{B_h} = \tan^{-1} \left[ \frac{(\mu_0 \mu / 2\pi r^3) \sin \lambda_m}{(\mu_0 \mu / 4\pi r^3) \cos \lambda_m} \right] = \tan^{-1} (2 \tan \lambda_m)$$

$$\Rightarrow \phi_i = \tan^{-1} (2 \tan 0^\circ) = \boxed{0^\circ}$$

b) At  $\lambda_m = 60^\circ$ 

$$B_h = 3.095 \times 10^{-5} T \cos 60^\circ = 1.55 \times 10^{-5} T$$

$$B_v = 6.190 \times 10^{-5} T \sin 60^\circ = 5.36 \times 10^{-5} T$$

$$B = \sqrt{B_h^2 + B_v^2} = \sqrt{(1.55 \times 10^{-5} T)^2 + (5.36 \times 10^{-5} T)^2}$$

$$= \boxed{5.58 \times 10^{-5} T}$$

$$\phi_i = \tan^{-1} (2 \tan \lambda_m) = \tan^{-1} (2 \tan 60^\circ) = \boxed{73.90^\circ}$$

c) At  $\lambda_m = 90^\circ$ ,

$$B_h = 3.095 \times 10^{-5} T \cos 90^\circ = 0$$

$$B_v = 6.190 \times 10^{-5} T \sin 90^\circ = 6.190 \times 10^{-5} T$$

So  $B = 6.190 \times 10^{-5} T$  twice the equatorial value!

$$\phi_i = \tan^{-1}(2 \tan \lambda_m) = \tan^{-1}(2 \tan 90^\circ) = 90^\circ$$

(32-2) Iron ceases to be ferromagnetic at its Curie temperature,  $T_c = 1043 \text{ K} = 770^\circ \text{C}$ . If the surface temperature is  $10^\circ \text{C}$  and the temperature increases by  $30^\circ \text{C}/\text{km}$ , iron will cease to be ferromagnetic at a depth of

$$d = \frac{(770^\circ \text{C} - 10^\circ \text{C})}{30^\circ \text{C}/\text{km}} = 25.3 \text{ km}$$

(32-3) First find the displacement current from

$$B = \frac{\mu_0 i_d}{2\pi r} \quad R = 3 \text{ mm} \quad \begin{array}{l} \text{plate} \\ \text{area} \\ \pi R^2 \end{array}$$

$$\Rightarrow i_d = \frac{2\pi r B}{\mu_0}$$

The displacement current is  $i_d = \epsilon_0 A \frac{dE}{dt}$

so

$$\epsilon_0 A \frac{dE}{dt} = \frac{2\pi r B}{\mu_0}$$

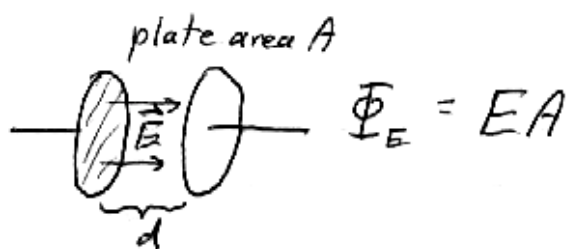
$$\frac{dE}{dt} = \frac{2\pi r B}{\mu_0 \epsilon_0 A} = \frac{2\pi r B}{\mu_0 \epsilon_0 \pi R^2} = \frac{2r B}{\mu_0 \epsilon_0 R^2}$$

Thus

$$\frac{dE}{dt} = \frac{2(6 \times 10^{-3} \text{ m})(2 \times 10^{-7} \text{ T})}{(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}})(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2})(3 \times 10^{-3} \text{ m})^2}$$

$$= 2.40 \times 10^{13} \frac{\text{N/C}}{\text{sec}}$$

$$(32-4) \quad \begin{aligned} \dot{\lambda}_d &= \epsilon_0 \frac{d\Phi_E}{dt} \\ &= \epsilon_0 A \frac{dE}{dt} \end{aligned}$$



If the voltage between the plates is  $V$ ,  
then

$$V = Ed \quad \text{or} \quad E = \frac{V}{d}$$

Then

$$\dot{\lambda}_d = \epsilon_0 A \frac{d}{dt} \left( \frac{V}{d} \right)$$

$$\dot{\lambda}_d = \epsilon_0 A \frac{1}{d} \frac{dV}{dt} \quad (\text{we seek } \frac{dV}{dt})$$

But  $\frac{\epsilon_0 A}{d} = C$ , the capacitance of a parallel plate capacitor. So

$$\dot{\lambda}_d = C \frac{dV}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{\dot{\lambda}_d}{C} = \frac{1.5 \text{ A}}{2 \times 10^{-6} \text{ F}} = \boxed{7.5 \times 10^5 \frac{\text{V}}{\text{sec}}}$$

is the rate at which the voltage must be changed

$$(32-5) \quad \text{a) } \dot{\lambda}_d = \epsilon_0 A \frac{dE}{dt}$$

$$= \epsilon_0 A \frac{d}{dt} \left[ (4 \times 10^5) \frac{\text{N}}{\text{C}} - (6 \times 10^4) \frac{\text{N/C}}{\text{sec}} t \right]$$

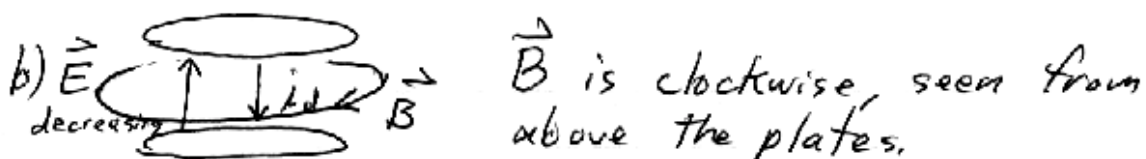
$$= -\epsilon_0 A (6 \times 10^4) \frac{\text{N/C}}{\text{sec}}$$

$$= -(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}) (4 \times 10^{-2} \text{ m}^2) (6 \times 10^4 \frac{\text{N}}{\text{C} \cdot \text{sec}})$$

$$= \boxed{-2.12 \times 10^{-8} \text{ A}}$$

Since  $\frac{dE}{dt} < 0$ , (+) charge is running off the positive plate

$\Rightarrow \dot{\lambda}_d$  is in the opposite direction of  $\vec{E}$



$$(33-6) \quad a) \quad \bar{I} = \frac{\mathcal{E}}{X_C} = \frac{\mathcal{E}}{1/\omega C} = \mathcal{E} \omega C$$

$$= (30 \text{ V})(2\pi)(10^3 \frac{1}{\text{sec}})(1.5 \times 10^{-6} \text{ F})$$

$$= \boxed{0.283 \text{ A}}$$

$$b) \quad I = \mathcal{E} \omega C = (30 \text{ V})(2\pi)(8 \times 10^3 \frac{1}{\text{sec}})(1.5 \times 10^{-6} \text{ F}) = \boxed{2.26 \text{ A}}$$

$$(33-7) \quad a) \quad I = \frac{\mathcal{E}}{X_L} = \frac{\mathcal{E}}{\omega L} = \frac{30 \text{ V}}{2\pi(10^3 \frac{1}{\text{sec}})(50 \times 10^{-3} \text{ H})} = \boxed{9.55 \times 10^{-2} \text{ A}}$$

$$b) \quad \bar{I} = \frac{\mathcal{E}}{\omega L} = \frac{30 \text{ V}}{2\pi(8 \times 10^3 \frac{1}{\text{sec}})(50 \times 10^{-3} \text{ H})} = \boxed{1.19 \times 10^{-2} \text{ A}}$$

$$(33-8) \quad \text{At resonance, } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1 \text{ H})(10^{-6} \text{ F})}} = 10^3 \frac{\text{rad}}{\text{sec}}$$

The impedance at resonance is

$$Z = \sqrt{R^2 + (\omega_0 L - \frac{1}{\omega_0 C})^2} = R \quad (\text{since } \omega_0 L = \frac{1}{\omega_0 C})$$

so

$$Z = 10 \Omega$$

The current amplitude is, at resonance,

$$I = \frac{\mathcal{E}}{Z} = \frac{10 \text{ V}}{10 \Omega} = 1 \text{ A}$$

The amplitude of the voltage across the inductor at resonance is

$$V_L = I X_L = I \omega_0 L = (1 \text{ A})(10^3 \frac{\text{rad}}{\text{sec}})(1 \text{ H})$$

$$= \boxed{10^3 \text{ V}} !$$

$$(33-9) \quad \tan \phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow R = \frac{\omega L - \frac{1}{\omega C}}{\tan \phi}$$

$$= \frac{2\pi(930 \frac{1}{\text{sec}})(88 \times 10^{-3} \text{H}) - \frac{1}{(2\pi)(930 \frac{1}{\text{sec}})(0.94 \times 10^{-6} \text{F})}}{\tan 75^\circ}$$

$$= \boxed{89 \Omega}$$

$$(33-10) \text{ a) } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_L^2} \quad \text{since } X_C = 0$$

$$Z = \sqrt{(12 \Omega)^2 + (1.3 \Omega)^2} = 12.07 \Omega$$

$$\text{b) } P_{\text{ave}} = E_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$= E_{\text{rms}} \frac{E_{\text{rms}}}{Z} \frac{R}{Z}$$

$$= \left(\frac{E_{\text{rms}}}{Z}\right)^2 R = I_{\text{rms}}^2 R$$

$$\text{So } P_{\text{ave}} = \left(\frac{120 \text{V}}{12.07 \Omega}\right)^2 (12.07 \Omega)$$

$$= \boxed{1.19 \times 10^3 \text{ W}}$$

$$(33-11) \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_L^2}$$

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{420 \text{V}}{\sqrt{(32 \Omega)^2 + (45 \Omega)^2}} = \boxed{7.61 \text{ A}}$$

$$(33-12) \quad V_s = V_p \frac{N_s}{N_p} = (100 \text{V}) \left(\frac{500}{50}\right) = \boxed{1000 \text{ V}}$$