Homework #7

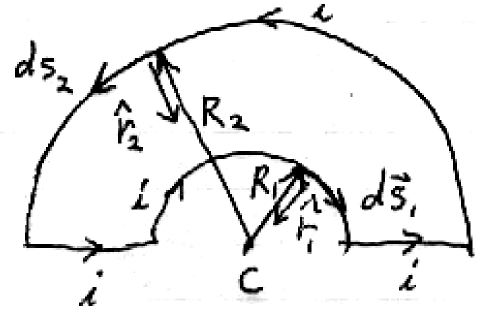
(30-1) The current of the electron beam is

$$i = 5.6 \times 10^{14} \frac{\text{electrons}}{\text{sec}} \times \frac{1.6\times10^{-19}\,\text{C}}{\text{electron}}$$

$$= 8.96 \times 10^{-5}\,\text{A}$$

So $$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}})(8.96\times10^{-5}\,\text{A})}{2\pi\,(1.5\times10^{-3}\,\text{m})} = \boxed{1.19\times10^{-8}\,\text{T}}$$

a)



The straight segments do not contribute to the magnetic field at C, since $\sin\theta = 0$ at left and $\sin\theta = 0$ at right

Use $$B = \frac{\mu_0 i}{4\pi}\int \frac{ds\,\sin\theta}{r^2}$$ and define "out of the page" to be the positive direction.

For the small arc, B is negative at point C ("into the page")

$$B_{small} = -\frac{\mu_0 i}{4\pi}\int \frac{ds\,\sin 90^\circ}{R_1^2} = -\frac{\mu_0 i}{4\pi R_1^2}\int ds$$

$$B_{small} = -\frac{\mu_0 i}{4\pi R_1^2}\left(\frac{1}{2}\right)(2\pi R_1) = -\frac{\mu_0 i}{4R_1}$$

For the large arc, B is positive at point C ("out of the page")

$$B_{large} = \frac{\mu_0 i}{4\pi}\int \frac{ds\,\sin 90^\circ}{4\pi R_2^2} = \frac{\mu_0 i}{4\pi R_2^2}\int ds$$

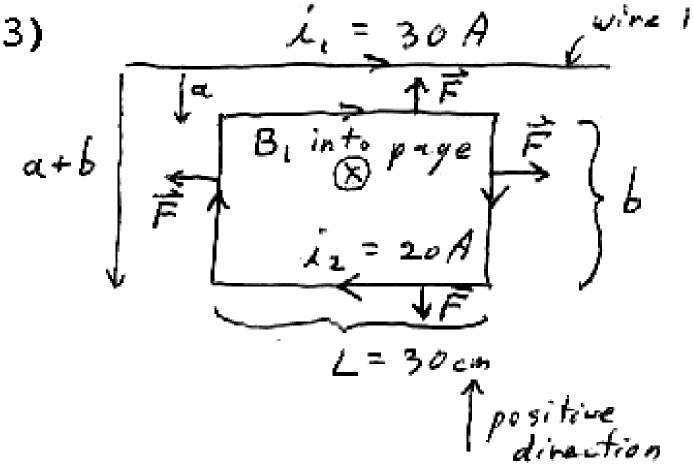
$$B_{large} = \frac{\mu_0 i}{4\pi R_2^2}\left(\frac{1}{2}\right)(2\pi R_2) = \frac{\mu_0 i}{4R_2}$$

So $$B_{total} = B_{small} + B_{large} = -\frac{\mu_0 i}{4R_1} + \frac{\mu_0 i}{4R_2}$$

or $B_{total} = \frac{\mu_0 i}{4}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \boxed{\frac{\mu_0 i}{4}\left(\frac{R_1 - R_2}{R_1 R_2}\right)}$

Because $R_1 < R_2$, B_{total} is negative ("into the page")

(30-3)



Below the top wire, its magnetic field is into the page.

The top wire (wire 1) will attract the top side of the loop, and repel the bottom side of the loop.

The forces on the left and right sides of the loop are in opposite directions. They will cancel.

So the force on the top side of the loop is

$$F_{top} = i_2 L B_1 \sin 90° = i_2 L \frac{\mu_0 i_1}{2\pi a} = \frac{\mu_0 i_1 i_2 L}{2\pi a}$$

The force on the bottom side of the loop is

$$F_{bottom} = -i_2 L B_1 \sin 90° = -i_2 L \frac{\mu_0 i_1}{2\pi(a+b)} = \frac{-\mu_0 i_1 i_2 L}{2\pi(a+b)}$$

So the net force on the loop is

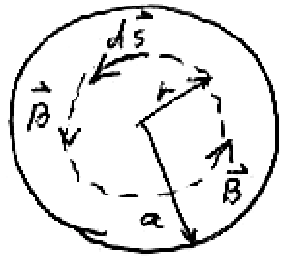
$$F_{net} = F_1 + F_2 = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+b}\right)$$

$$F_{net} = \frac{(4\pi \times 10^{-7}\ \frac{T\cdot m}{A})(30\ A)(20\ A)(0.3\ m)}{2\pi}\left(\frac{1}{0.01\ m} - \frac{1}{(0.01\ m + 0.08\ m)}\right)$$

$$= \boxed{3.2 \times 10^{-3}\ N} \text{ toward wire 1}$$

(30-4) Assume that the current is coming out of the page.

For $r<a$



The current density is

$$J = \frac{i}{\pi a^2}$$

$$\int B\,ds\cos\theta = \mu_0 i_{enc} \quad \text{with } i_{enc} = J(\pi r^2) \text{ and } \theta = 0°$$

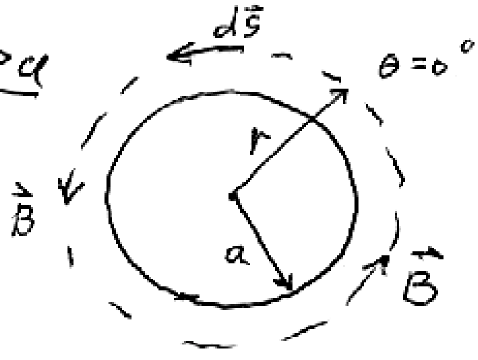
So $$\int B\,ds\cos 0° = \mu_0\left(\frac{i}{\pi a^2}\right)(\pi r^2)$$

$$B\int ds = \mu_0 i \frac{r}{a}$$

$$B\,2\pi r = \mu_0 i \frac{r^2}{a^2}$$

$$\boxed{B = \mu_0 i \frac{r}{2\pi a^2}} \quad \text{for } r<a$$

For $r>a$



$$\int B\,ds\underbrace{\cos\theta}_{=1} = \mu_0 i$$

$$B\int ds = \mu_0 i$$

$$B\,2\pi r = \mu_0 i$$

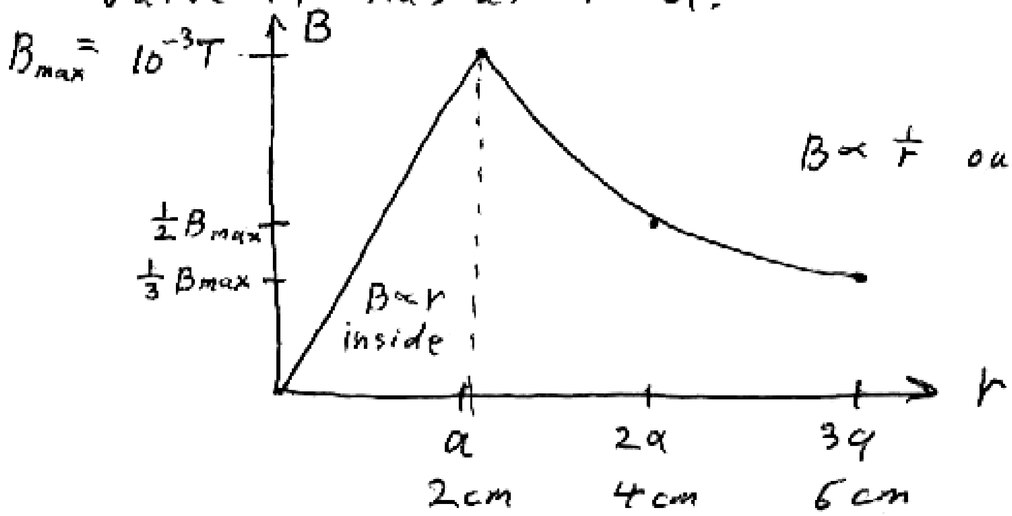
$$\boxed{B = \mu_0 i \frac{1}{2\pi r}} \quad \text{for } r>a$$

Note that the two formulas agree for $r=a$!

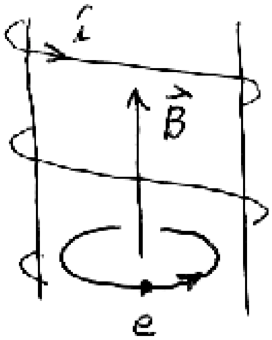
At $r=a$, $$B = \mu_0 i \frac{1}{2\pi a} = \frac{(4\pi\times10^{-7}\,\frac{T\cdot m}{A})(100A)}{2\pi(0.02m)}$$

$$\boxed{B = 10^{-3}\,T \text{ at } r=a}$$

At $r = 6\,cm = 3a$, B will be $\frac{1}{3}$ of the value it has at $r = a$.



(30-5) Inside the solenoid, $B = \mu_0 n i$



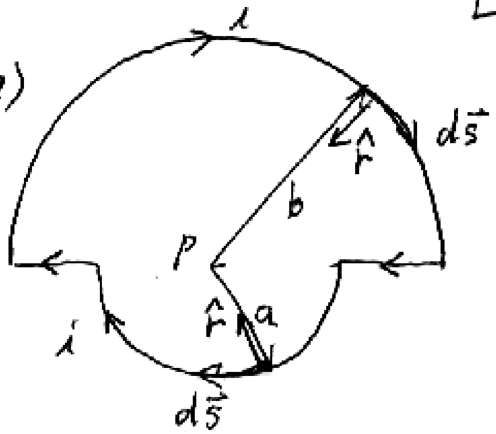
Using $r = \dfrac{mv}{|q|B} = \dfrac{mv}{|q|\mu_0 n i}$

$\Rightarrow i = \dfrac{mv}{|q|\mu_0 n r}$

$$i = \frac{(9.11\times10^{-31}\,kg)(0.046)(3\times10^{8}\,m/sec)}{(1.6\times10^{-19}\,C)(4\pi\times10^{-7}\,\frac{T\cdot m}{A})(100\,\frac{1}{cm})(\frac{100\,cm}{1\,m})(0.023\,m)}$$

$= \boxed{0.272\,A}$

(30-6) a)



The straight segments do not contribute to the magnetic field at P.

Both arcs produce a magnetic field that is <u>into the page</u> at P.

Use $B = \dfrac{\mu_0 i}{4\pi}\displaystyle\int \frac{ds \sin\theta}{r^2}$ for the arcs

For the small arc,

$$B_{small} = \frac{\mu_0 i}{4\pi} \int \frac{ds \sin 90^0}{a^2} = \frac{\mu_0 i}{4\pi a^2} \int ds$$

$$B_{small} = \frac{\mu_0 i}{4\pi a^2} \left(\frac{1}{2}\right)(2\pi a) = \frac{\mu_0 i}{4a}$$

For the large arc, we get $B_{large} = \frac{\mu_0 i}{4b}$

So $B_{total} = B_{small} + B_{large} = \frac{\mu_0 i}{4a} + \frac{\mu_0 i}{4b}$

$$\boxed{B_{total} = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right)}$$ into the page

b) $\mu = i \times$ (loop area)

$$= i\left[\frac{1}{2}(\pi a^2) + \frac{1}{2}(\pi b^2)\right] = \boxed{\frac{i\pi}{2}(a^2 + b^2)}$$

(31-7) a) $\mathcal{E} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\left[\left(6.0\,\frac{mWb}{sec^2}\right)t^2 + \left(7.0\,\frac{mWb}{sec}\right)t\right]$

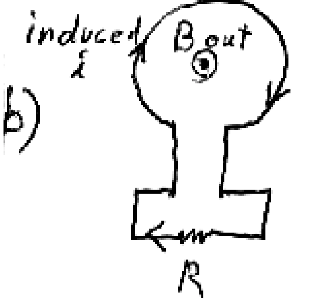
$$= -\left(12\,\frac{mWb}{sec^2}\,t + 7\,\frac{mWb}{sec}\right)$$

At $t = 2$ sec (and using 1 volt $= 1\,\frac{Wb}{sec}$)

$$\mathcal{E} = -\left[\left(12\,\frac{mWb}{sec^2}\right)(2\,sec) + 7\,\frac{mWb}{sec}\right] = \boxed{-31\,mV}$$

The magnitude of $\mathcal{E}$ is $\boxed{31\,mV}$

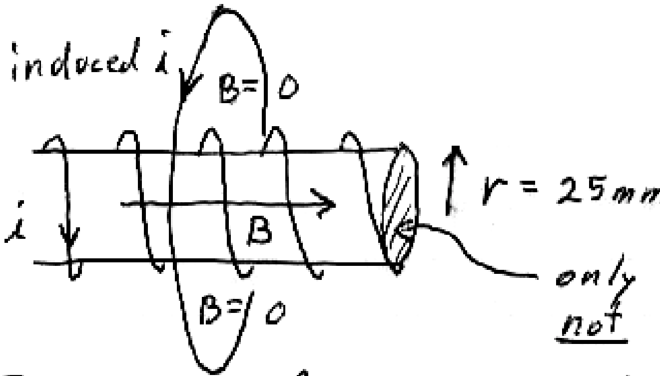
b)



Since B is getting stronger out of the page, the induced current will oppose this change by creating its own magnetic field into the page.

$\Rightarrow$ induced current flows <u>to the</u> <u>left</u> through R

(31-8)



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

For a uniform rate of change, we can use

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(BA)}{\Delta t} = -A\frac{\Delta B}{\Delta t}$$

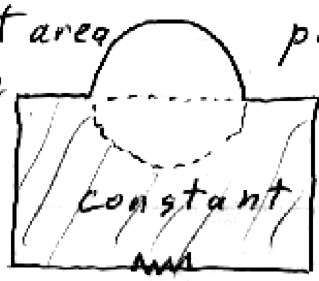
So $\mathcal{E} = -(\pi r^2)\frac{B_f - B_i}{\Delta t}$ with $B = \mu_0 n i$

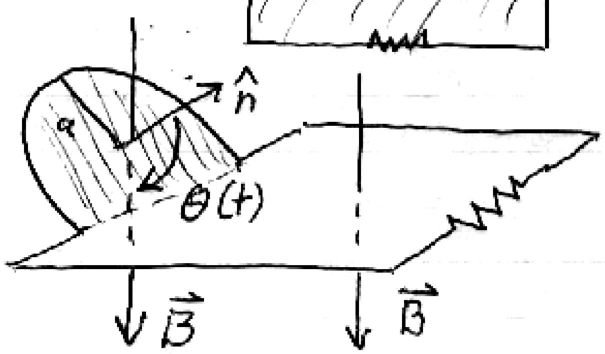
$$= -(\pi r^2)\frac{\mu_0 n(i_f - i_i)}{\Delta t}$$

$$= -\pi(0.025m)^2 \frac{(4\pi\times10^{-7}\frac{T\cdot m}{A})(10^4\frac{1}{m})(0.5A - 1A)}{(10\times10^{-3}sec)}$$

$$= \boxed{1.23\times10^{-3}\ V}$$

(31-9) The loop area consists of a constant part plus a rotating semicircular part. The flux through the constant area part does not change, so we only have to consider the rotating semicircle.





Using $\theta(t) = \omega t = 2\pi f t$, the flux through the loop at time t is

$$\Phi_B = BA\cos\theta$$

$$= B\frac{\pi a^2}{2}\cos 2\pi f t$$

So $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{1}{2}B\pi a^2 \frac{d}{dt}\cos 2\pi f t$

$$\mathcal{E} = \tfrac{1}{2} B \pi a^2 (2\pi f) \sin(2\pi f t)$$

$$= \boxed{B \pi^2 a^2 f \sin(2\pi f t)}$$

The frequency of the current is $\boxed{f}$

(31-10) $i = i_0 e^{-Rt/L}$ so $\frac{i}{i_0} = e^{-Rt/L}$

Therefore $\ln \frac{i}{i_0} = -\frac{Rt}{L}$

$$R = -\frac{L}{t} \ln \frac{i}{i_0}$$

$$= -\frac{10\,H}{1\,sec} \ln\left(\frac{10^{-2} A}{1\,A}\right)$$

$$= \boxed{46.1\,\Omega}$$

(31-11) a) $U_L = \frac{1}{2} L i^2$ so $L = \frac{2U_L}{i^2} = \frac{2(0.025\,J)}{(0.06\,A)^2} = \boxed{13.9\,H}$

b) $\frac{U_L'}{U_L} = \frac{\frac{1}{2}L(i')^2}{\frac{1}{2}Li^2} = \left(\frac{i'}{i}\right)^2 = 4$

So $i' = 2i = 2(60\,mA) = \boxed{120\,mA}$

(31-12) a) $B = \mu_0 n i = \mu_0 \left(\frac{N}{L}\right) i = (4\pi \times 10^{-7} \frac{T \cdot m}{A})(\frac{950}{0.85\,m})(6.6\,A)$

$$= 9.27 \times 10^{-3}\,T$$

$$u_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2(4\pi \times 10^{-7} \frac{T \cdot m}{A})} (9.27 \times 10^{-3}\,T)^2$$

$$= \boxed{34.2\,\frac{J}{m^3}}$$

b) $U_B = u_B \times \text{volume} = (34.2\,\frac{J}{m^3})(0.85\,m)(17\,cm^2)\left(\frac{1\,m}{100\,cm}\right)^2$

$$= \boxed{4.94 \times 10^{-2}\,J}$$