

Homework #4

$$(26-1) \quad C = \frac{\epsilon_0 A}{d} \quad \text{so} \quad d = \frac{\epsilon_0 A}{C}$$

$$= \frac{(8.85 \times 10^{-12} \frac{C^2}{Nm^2})(1m^2)}{1F}$$

$$= \boxed{8.85 \times 10^{-12} m}$$

This is smaller than an atom ( $\sim 10^{-10} m$  across), so this cannot be constructed.

$$(26-2) \quad a) \quad C = 4\pi\epsilon_0 \frac{ab}{b-a} = \frac{1}{k} \frac{ab}{b-a}$$

$$\text{So } C = \frac{1}{8.99 \times 10^9 \frac{Nm^2}{C^2}} \frac{(38 \times 10^{-3} m)(40 \times 10^{-3} m)}{(40 \times 10^{-3} m - 38 \times 10^{-3} m)}$$

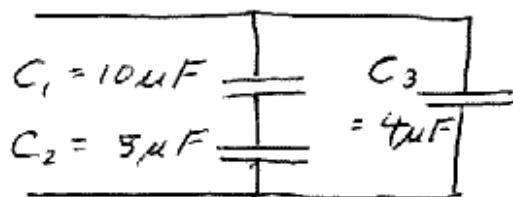
$$= \boxed{8.45 \times 10^{-11} F \text{ or } 84.5 pF}$$

$$b) \quad \text{Set this } C = \frac{\epsilon_0 A}{d} \text{ with } d = b - a = 2 \text{ mm}$$

$$\text{Then } A = \frac{Cd}{\epsilon_0} = \frac{(8.45 \times 10^{-11} F)(2 \times 10^{-3} m)}{8.85 \times 10^{-12} \frac{C^2}{Nm^2}}$$

$$= \boxed{1.91 \times 10^{-2} m^2}$$

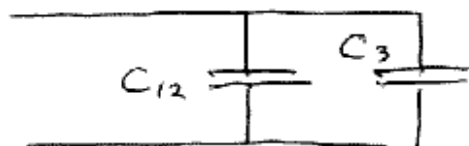
(26-3)



$C_1$  and  $C_2$  are in series,

$$\text{So } \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10 \mu F)(5 \mu F)}{10 \mu F + 5 \mu F}$$



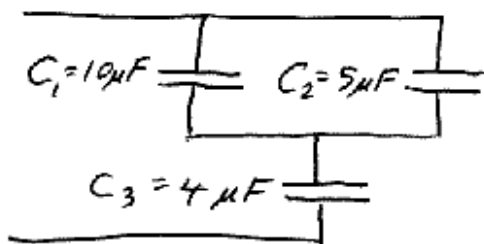
$$C_{12} = 3.33 \mu F$$

Now  $C_{12}$  and  $C_3$  are in

parallel, so

$$C_{123} = C_{12} + C_3 = 3.33 \mu F + 4 \mu F = \boxed{7.33 \mu F}$$

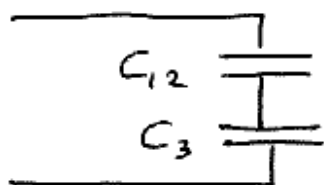
(26-4)



$C_1$  and  $C_2$  are in parallel,  
so

$$C_{12} = C_1 + C_2 = 10\mu F + 5\mu F$$

$$C_{12} = 15\mu F$$

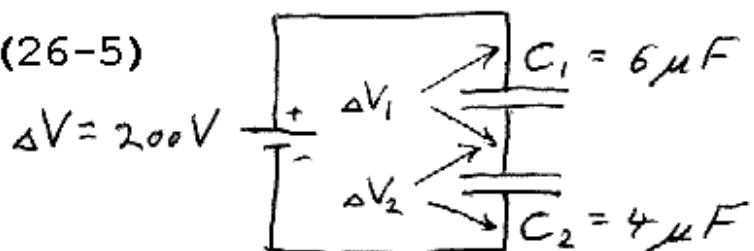


$C_{12}$  and  $C_3$  are in series, so

$$C_{123} = \frac{C_{12} C_3}{C_{12} + C_3} = \frac{(15\mu F)(4\mu F)}{15\mu F + 4\mu F}$$

$$= \boxed{3.16\mu F}$$

(26-5)



a)  $C_1$  and  $C_2$  are in series,  
so

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{12} = \frac{(6\mu F)(4\mu F)}{6\mu F + 4\mu F}$$

$$C_{12} = \boxed{2.4\mu F}$$

$$b) Q = C_{12} \Delta V = (2.4 \times 10^{-6} F)(200 V) = 4.8 \times 10^{-4} C$$

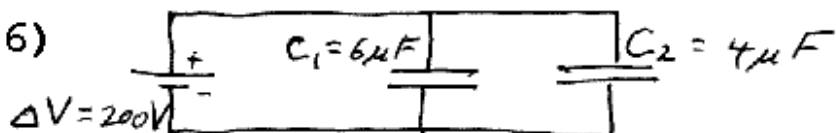
Each of the capacitors has this charge

$$c) \Delta V_1 = \frac{Q}{C_1} = \frac{4.8 \times 10^{-4} C}{6 \times 10^{-6} F} = \boxed{80 V}$$

$$\Delta V_2 = \frac{Q}{C_2} = \frac{4.8 \times 10^{-4} C}{4 \times 10^{-6} F} = \boxed{120 V}$$

note that  
these add  
up to 200V

(26-6)



a)  $C_1$  and  $C_2$  are in series,  
so

$$C_{12} = C_1 + C_2 = 6\mu F + 4\mu F = \boxed{10\mu F}$$

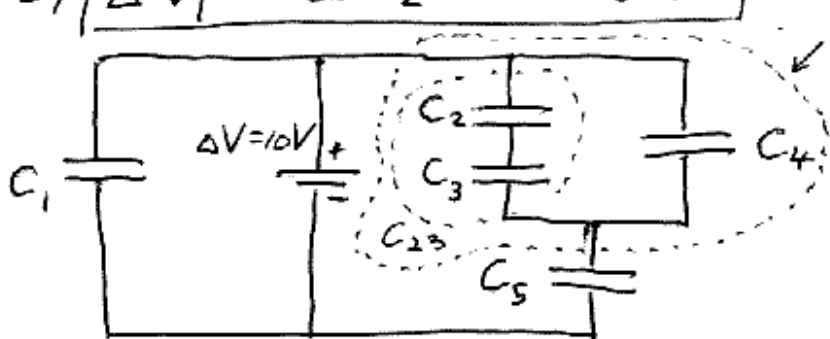
b) The voltage difference across each capacitor is  $\Delta V = 200 \text{ V}$ . So

$$Q_1 = C_1 \Delta V = (6 \times 10^{-6} \text{ F})(200 \text{ V}) = \boxed{1.2 \times 10^{-3} \text{ C}}$$

$$Q_2 = C_2 \Delta V = (4 \times 10^{-6} \text{ F})(200 \text{ V}) = \boxed{8 \times 10^{-4} \text{ C}}$$

c)  $\Delta V_1 = \Delta V_2 = 200 \text{ V}$

(26-7)



a) There are  $\Delta V = 10 \text{ V}$  across  $C_1$ , so

$$Q_1 = C_1 \Delta V$$

$$Q_1 = (10^{-5} \text{ F})(10 \text{ V}) = \boxed{10^{-4} \text{ C}}$$

b) First, find the equivalent capacitance of capacitors 2, 3, 4, and 5.  $C_2$  and  $C_3$  are in series, so

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{(10 \mu\text{F})(10 \mu\text{F})}{10 \mu\text{F} + 10 \mu\text{F}}$$

$$C_{23} = 5 \mu\text{F}$$

Next,  $C_{23}$  and  $C_4$  are in parallel, so

$$C_{234} = C_{23} + C_4 = 5 \mu\text{F} + 10 \mu\text{F} = 15 \mu\text{F}$$

Finally,  $C_{234}$  and  $C_5$  are in series, so

$$C_{2345} = \frac{C_{234} C_5}{C_{234} + C_5} = \frac{(15 \mu\text{F})(10 \mu\text{F})}{15 \mu\text{F} + 10 \mu\text{F}} = 6 \mu\text{F}$$

The charge on these capacitors is

$$Q_{2345} = C_{2345} \Delta V = (6 \times 10^{-6} \text{ F})(10 \text{ V}) \\ = 6 \times 10^{-5} \text{ C}$$

Since  $C_{234}$  and  $C_5$  are in series, they each have this much charge:

$$Q_{234} = Q_5 = 6 \times 10^{-5} \text{ C}$$

The voltage across  $C_5$  is

$$\Delta V_5 = \frac{Q_5}{C_5} = \frac{6 \times 10^{-5} \text{ C}}{10^{-5} \text{ F}} = 6 \text{ V}$$

Since there is 10V across  $C_{2345}$ , the voltage across just  $C_{234}$  is

$$\Delta V_{234} = \Delta V_{2345} - \Delta V_5 = 10 \text{ V} - 6 \text{ V} = 4 \text{ V}$$

Since  $C_{23}$  and  $C_4$  are in parallel,

$$\Delta V_{23} = \Delta V_4 = 4 \text{ V}$$

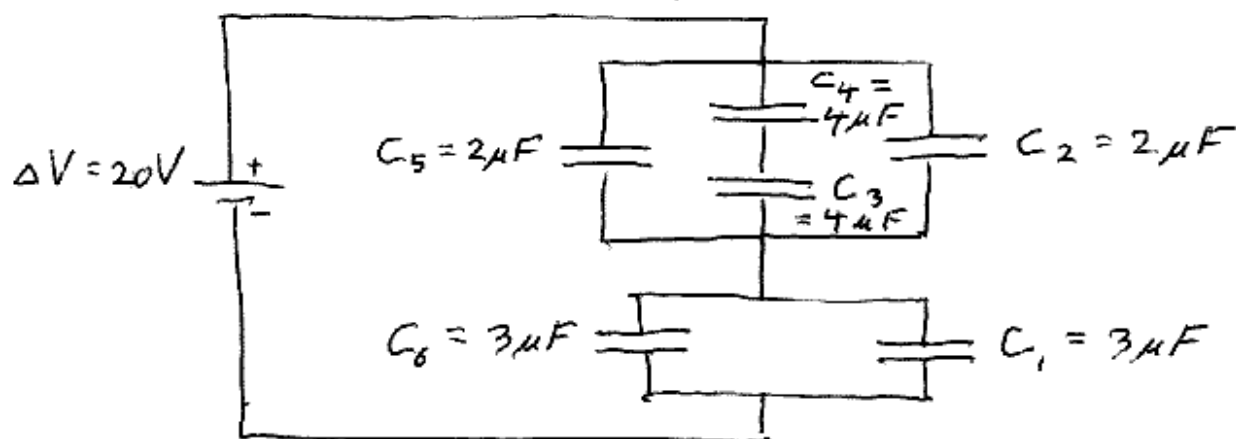
The charge on  $C_{23}$  is therefore

$$Q_{23} = C_{23} \Delta V_{23} = (5 \times 10^{-6} \text{ F})(4 \text{ V}) \\ = 2 \times 10^{-5} \text{ C}$$

Because  $C_2$  and  $C_3$  are in series, they each have charge  $Q_{23}$ . So

$$\boxed{Q_2 = 2 \times 10^{-5} \text{ C}}$$

(26-8) Start by redrawing the circuit as



a)  $C_3$  and  $C_4$  are in series, so

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4\mu\text{F})(4\mu\text{F})}{4\mu\text{F} + 4\mu\text{F}} = 2\mu\text{F}$$

$C_2$ ,  $C_{34}$ , and  $C_5$  are all in parallel, so

$$\begin{aligned} C_{2345} &= C_2 + C_{34} + C_5 \\ &= 2\mu\text{F} + 2\mu\text{F} + 2\mu\text{F} = 6\mu\text{F} \end{aligned}$$

Next,  $C_1$  and  $C_6$  are in parallel, so

$$C_{16} = C_1 + C_6 = 3\mu\text{F} + 3\mu\text{F} = 6\mu\text{F}$$

Since  $C_{2345}$  and  $C_{16}$  are in series,

$$\begin{aligned} C_{123456} &= \frac{C_{2345} C_{16}}{C_{2345} + C_{16}} = \frac{(6\mu\text{F})(6\mu\text{F})}{6\mu\text{F} + 6\mu\text{F}} \\ &= \boxed{3\mu\text{F}} \end{aligned}$$

$$\begin{aligned} \text{b) } Q_{123456} &= C_{123456} \Delta V = (3 \times 10^{-6} \text{F})(20 \text{V}) \\ &= \boxed{6 \times 10^{-5} \text{C}} \end{aligned}$$

- c) Since  $C_{2345}$  and  $C_{16}$  are in series they each have charge  $Q_{123456} = 6 \times 10^{-5} \text{ C}$ . Capacitors  $C_1$  and  $C_6$  are in parallel, so they share the charge  $Q_{16} = 6 \times 10^{-5} \text{ C}$ . That is,

$$\rightarrow Q_1 + Q_6 = 6 \times 10^{-5} \text{ C}$$

And because  $C_1$  and  $C_6$  are in parallel,

$$\Delta V_1 = \Delta V_6$$

Thus 
$$\frac{Q_1}{C_1} = \frac{Q_6}{C_6}$$

or 
$$\frac{Q_1}{Q_6} = \frac{C_1}{C_6} = \frac{3 \mu\text{F}}{3 \mu\text{F}} = 1$$

$$\Rightarrow Q_1 = Q_6$$

Substituting into here gives

$$Q_1 + Q_6 = 2Q_1 = 6 \times 10^{-5} \text{ C}$$

$$\Rightarrow \boxed{Q_1 = 3 \times 10^{-5} \text{ C}}$$

- d) Since  $Q_{2345} = 6 \times 10^{-5} \text{ C}$ , the voltage across  $C_{2345}$  is 
$$\Delta V_{2345} = \frac{Q_{2345}}{C_{2345}} = \frac{6 \times 10^{-5} \text{ C}}{6 \times 10^{-6} \text{ F}} = 10 \text{ V}$$

The charge on  $C_2$  is thus (using  $\Delta V_2 = \Delta V_{2345}$ )

$$Q_2 = C_2 \Delta V_2 = (2 \times 10^{-6} \text{ F})(10 \text{ V})$$

$$= \boxed{2 \times 10^{-5} \text{ C}}$$

e) Using  $\Delta V_{34} = \Delta V_{2345} = 10V$ , the charge on  $C_{34}$  is

$$Q_{34} = C_{34} \Delta V_{34} \\ = (2 \times 10^{-6} F)(10V) = 2 \times 10^{-5} C$$

Because  $C_3$  and  $C_4$  are in series, they each have this much charge, so

$$Q_3 = 2 \times 10^{-5} C$$

(26-9) a)  $C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \frac{C^2}{Nm^2})(40 \text{ cm}^2)(\frac{1m}{100 \text{ cm}})^2}{10^{-3} m}$

$$= 3.54 \times 10^{-11} F \text{ or } 35.4 \text{ pF}$$

b)  $Q = CV = (3.54 \times 10^{-11} F)(600V)$

$$= 2.12 \times 10^{-8} C$$

c)  $U = \frac{1}{2} CV^2 = \frac{1}{2} (3.54 \times 10^{-11} F)(600V)^2$

$$= 6.37 \times 10^{-6} J$$

d)  $E = \frac{\Delta V}{d} = \frac{600V}{10^{-3} m} = 6 \times 10^5 \frac{V}{m}$

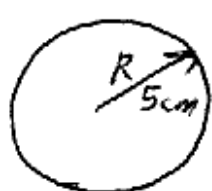
e)  $u = \frac{U}{V} = \frac{6.37 \times 10^{-6} J}{(40 \text{ cm}^2)(\frac{1m}{100 \text{ cm}})^2 (10^{-3} m)} = 1.59 \frac{J}{m^2}$

(26-10) a)  $u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left( \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} \right)^2$

$$u = \frac{e^2}{32\pi^2 \epsilon_0 r^4}$$

b) The energy density diverges ( $\rightarrow \infty$ ) as  $r \rightarrow 0$ .

(26-11) At the surface of the sphere,



$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \text{ and } V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = 8000 \text{ V}$$

$$\text{So } E = \frac{V}{R} = \frac{8000 \text{ V}}{0.05 \text{ m}} = 1.6 \times 10^5 \frac{\text{V}}{\text{m}}$$

$$\text{Therefore } u = \frac{1}{2} \epsilon_0 E^2$$

$$= \frac{1}{2} (8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}) (1.6 \times 10^5 \frac{\text{V}}{\text{m}})^2$$

$$= \boxed{0.113 \frac{\text{J}}{\text{m}^3}}$$

(26-12) a)  $C = \frac{\epsilon_0 A}{d}$

$$\Rightarrow d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}) (0.35 \text{ m}^2)}{50 \times 10^{-12} \text{ F}}$$

$$= \boxed{6.20 \times 10^{-2} \text{ m}}$$

b)  $C = K C_{\text{air}} = 5.6 (50 \text{ pF}) = \boxed{280 \text{ pF}}$