Homework #4

(26-1) $C = \frac{\varepsilon_0 A}{d}$ so $d = \frac{\varepsilon_0 A}{C}$

$$= \frac{(8.85\times10^{-12}\,\frac{C^2}{Nm^2})(1\,m^2)}{1\,F}$$

$$= \boxed{8.85\times10^{-12}\,m}$$

This is smaller than an atom ($\sim 10^{-10}$ m across), so this cannot be constructed.

(26-2) a) $C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{1}{k}\frac{ab}{b-a}$

So $C = \frac{1}{8.99\times10^{9}\,\frac{Nm^2}{C^2}} \frac{(38\times10^{-3}m)(40\times10^{-3}m)}{(40\times10^{-3}m - 38\times10^{-3}m)}$

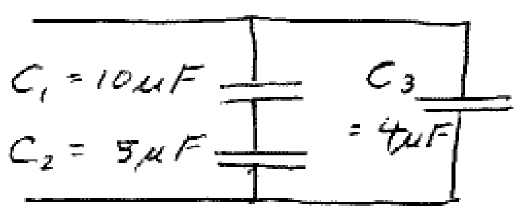
$$= \boxed{8.45\times10^{-11}\,F \text{ or } 84.5\,pF}$$

b) Set this $C = \frac{\varepsilon_0 A}{d}$ with $d = b - a = 2\,mm$

Then $A = \frac{Cd}{\varepsilon_0} = \frac{(8.45\times10^{-11}F)(2\times10^{-3}m)}{8.85\times10^{-12}\,C^2/Nm^2}$

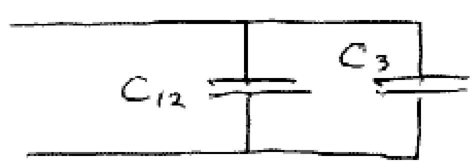
$$= \boxed{1.91\times10^{-2}\,m^2}$$

(26-3)



C_1 and C_2 are in series, so $\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$

$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10\mu F)(5\mu F)}{10\mu F + 5\mu F}$

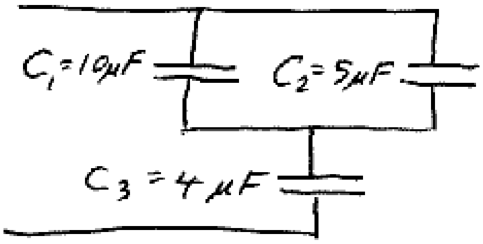


$C_{12} = 3.33\mu F$

Now C_{12} and C_3 are in parallel, so

$C_{123} = C_{12} + C_3 = 3.33\mu F + 4\mu F = \boxed{7.33\mu F}$

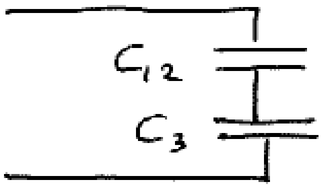
(26-4)



C_1 and C_2 are in parallel, so

$$C_{12} = C_1 + C_2 = 10\mu F + 5\mu F$$

$$C_{12} = 15\mu F$$

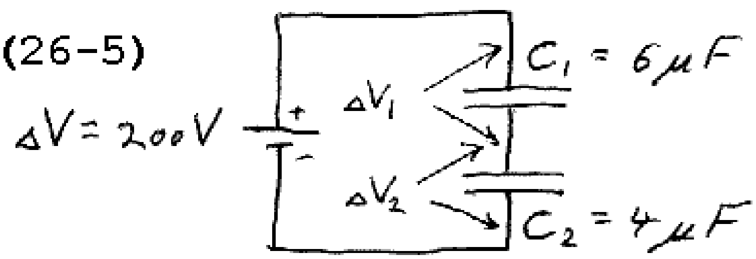


C_{12} and C_3 are in series, so

$$C_{123} = \frac{C_{12}\,C_3}{C_{12}+C_3} = \frac{(15\mu F)(4\mu F)}{15\mu F + 4\mu F}$$

$$= \boxed{3.16\,\mu F}$$

(26-5)



a) C_1 and C_2 are in series, so

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{12} = \frac{(6\mu F)(4\mu F)}{6\mu F + 4\mu F}$$

$$C_{12} = \boxed{2.4\mu F}$$

b) $Q = C_{12}\Delta V = (2.4\times10^{-6}F)(200V) = 4.8\times10^{-4}C$

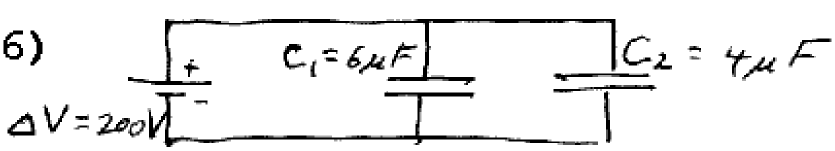
Each of the capacitors has this charge

c) $\Delta V_1 = \frac{Q}{C_1} = \frac{4.8\times10^{-4}C}{6\times10^{-6}F} = \boxed{80\,V}$

$\Delta V_2 = \frac{Q}{C_2} = \frac{4.8\times10^{-4}C}{4\times10^{-6}\;C} = \boxed{120\,V}$

note that these add up to 200 V

(26-6)



a) C_1 and C_2 are in series, so

$$C_{12} = C_1 + C_2 = 6\mu F + 4\mu F = \boxed{10\mu F}$$

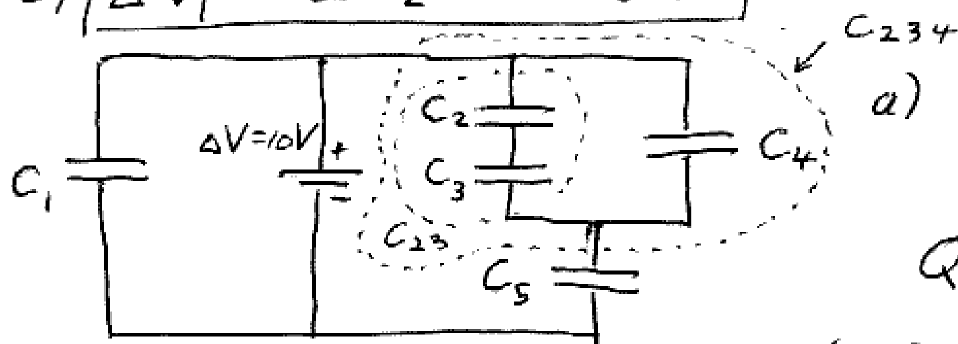
b) The voltage difference across each capacitor is $\Delta V = 200\,V$. So

$$Q_1 = C_1 \Delta V = (6\times10^{-6}F)(200V) = \boxed{1.2\times10^{-3}C}$$

$$Q_2 = C_2 \Delta V = (4\times10^{-6}F)(200V) = \boxed{8\times10^{-4}C}$$

c) $\boxed{\Delta V_1 = \Delta V_2 = 200\,V}$

(26-7)



a) There are $\Delta V = 10V$ across C_1, so

$$Q_1 = C_1 \Delta V$$

$$Q_1 = (10^{-5}F)(10V) = \boxed{10^{-4}C}$$

b) First, find the equivalent capacitance of capacitors 2, 3, 4, and 5. C_2 and C_3 are in series, so

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{(10\mu F)(10\mu F)}{10\mu F + 10\mu F}$$

$$C_{23} = 5\mu F$$

Next, C_{23} and C_4 are in parallel, so

$$C_{234} = C_{23} + C_4 = 5\mu F + 10\mu F = 15\mu F$$

Finally, C_{234} and C_5 are in series, so

$$C_{2345} = \frac{C_{234}C_5}{C_{234} + C_5} = \frac{(15\mu F)(10\mu F)}{15\mu F + 10\mu F} = 6\mu F$$

The charge on these capacitors is

$$Q_{2345} = C_{2345}\Delta V = (6\times10^{-6}F)(10V)$$
$$= 6\times10^{-5}C$$

Since C_{234} and C_5 are in series, they <u>each</u> have this much charge:

$$Q_{234} = Q_5 = 6\times10^{-5}C$$

The voltage across C_5 is

$$\Delta V_5 = \frac{Q_5}{C_5} = \frac{6\times10^{-5}C}{10^{-5}F} = 6V$$

Since there is 10V across C_{2345}, the voltage across just C_{234} is

$$\Delta V_{234} = \Delta V_{2345} - \Delta V_5 = 10V - 6V = 4V$$

Since C_{23} and C_4 are in parallel,

$$\Delta V_{23} = \Delta V_4 = 4V$$

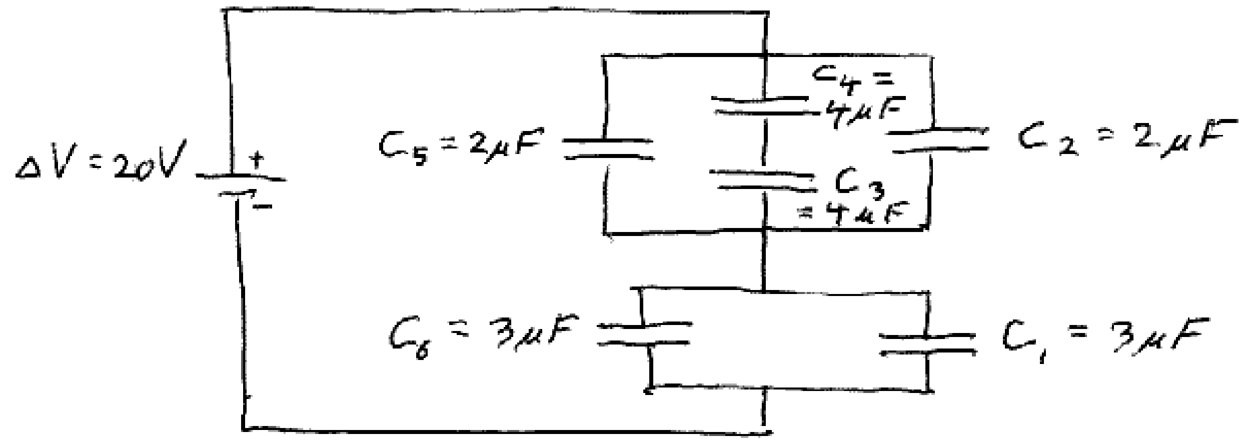
The charge on C_{23} is therefore

$$Q_{23} = C_{23}\Delta V_{23} = (5\times10^{-6}F)(4V)$$
$$= 2\times10^{-5}C$$

Because C_2 and C_3 are in series, they <u>each</u> have charge Q_{23}. So

$$\boxed{Q_2 = 2\times10^{-5}C}$$

(26-8) Start by redrawing the circuit as



a) C_3 and C_4 are in series, so

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4\mu F)(4\mu F)}{4\mu F + 4\mu F} = 2\mu F$$

C_2, C_{34}, and C_5 are all in parallel, so

$$C_{2345} = C_2 + C_{34} + C_5$$

$$= 2\mu F + 2\mu F + 2\mu F = 6\mu F$$

Next, C_1 and C_6 are in parallel, so

$$C_{16} = C_1 + C_6 = 3\mu F + 3\mu F = 6\mu F$$

Since C_{2345} and C_{16} are in series,

$$C_{123456} = \frac{C_{2345} C_{16}}{C_{2345} + C_{16}} = \frac{(6\mu F)(6\mu F)}{6\mu F + 6\mu F}$$

$$= \boxed{3\mu F}$$

b) $Q_{123456} = C_{123456} \Delta V = (3 \times 10^{-6} F)(20 V)$

$$= \boxed{6 \times 10^{-5} C}$$

c) Since C_{2345} and C_{16} are in series they each have charge $Q_{123456} = 6 \times 10^{-5}$ C. Capacitors C_1 and C_6 are in parallel, so they share the charge $Q_{16} = 6 \times 10^{-5}$ C. That is,

$$Q_1 + Q_6 = 6 \times 10^{-5} C$$

And because C_1 and C_6 are in parallel,

$$\Delta V_1 = \Delta V_6$$

Thus
$$\frac{Q_1}{C_1} = \frac{Q_6}{C_6}$$

or
$$\frac{Q_1}{Q_6} = \frac{C_1}{C_6} = \frac{3\mu F}{3\mu F} = 1$$

$$\Rightarrow Q_1 = Q_6$$

Substituting into here gives

$$Q_1 + Q_6 = 2Q_1 = 6 \times 10^{-5} C$$

$$\Rightarrow \boxed{Q_1 = 3 \times 10^{-5} C}$$

d) Since $Q_{2345} = 6 \times 10^{-5}$ C, the voltage across C_{2345} is

$$\Delta V_{2345} = \frac{Q_{2345}}{C_{2345}} = \frac{6 \times 10^{-5} C}{6 \times 10^{-6} F} = 10 V$$

The charge on C_2 is thus (using $\Delta V_2 = \Delta V_{2345}$)

$$Q_2 = C_2 \Delta V_2 = (2 \times 10^{-6} F)(10 V)$$

$$= \boxed{2 \times 10^{-5} C}$$

e) Using $\Delta V_{34} = \Delta V_{2345} = 10\,V$, the charge on C_{34} is

$$Q_{34} = C_{34}\,\Delta V_{34}$$

$$= (2\times 10^{-6}\,F)(10\,V) = 2\times 10^{-5}\,C$$

Because C_3 and C_4 are in series, they each have this much charge, so

$$\boxed{Q_3 = 2\times 10^{-5}\,C}$$

(26-9) a) $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times 10^{-12}\,\frac{C^2}{Nm^2})(40\,cm^2)(\frac{1m}{100\,cm})^2}{10^{-3}\,m}$

$$= \boxed{3.54\times 10^{-11}\,F \text{ or } 35.4\,pF}$$

b) $Q = C\Delta V = (3.54\times 10^{-11}\,F)(600\,V)$

$$= \boxed{2.12\times 10^{-8}\,C}$$

c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(3.54\times 10^{-11}\,F)(600\,V)^2$

$$= \boxed{6.37\times 10^{-6}\,J}$$

d) $E = \frac{\Delta V}{d} = \frac{600\,V}{10^{-3}\,m} = \boxed{6\times 10^{5}\,\frac{V}{m}}$

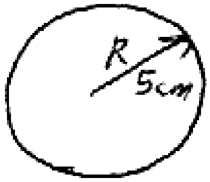
e) $u = \frac{U}{V} = \frac{6.37\times 10^{-6}\,J}{(40\,cm^2)(\frac{1m}{100\,cm})^2(10^{-3}\,m)} = \boxed{1.59\,\frac{J}{m^2}}$

(26-10) a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}\right)^2$

$$\boxed{u = \frac{e^2}{32\pi^2\varepsilon_0 r^4}}$$

b) The energy density <u>diverges</u> ($\to \infty$) as $r \to 0$.

(26-11) At the surface of the sphere,



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \quad \text{and} \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 8000\,V$$

$$\text{So} \quad E = \frac{V}{R} = \frac{8000\,V}{0.05\,m} = 1.6 \times 10^{5}\,\frac{V}{m}$$

$$\text{Therefore} \quad u = \frac{1}{2}\varepsilon_0 E^2$$

$$= \frac{1}{2}\left(8.85\times10^{-12}\,\frac{C^2}{Nm^2}\right)\left(1.6\times10^{5}\,\frac{V}{m}\right)^2$$

$$= \boxed{0.113\ \frac{J}{m^3}}$$

(26-12) a) $C = \frac{\varepsilon_0 A}{d}$

$$\Rightarrow d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\,\frac{C^2}{Nm^2}\right)\left(0.35\,m^2\right)}{50\times10^{-12}\,F}$$

$$= \boxed{6.20\times10^{-2}\,m}$$

b) $C = K\,C_{air} = 5.6\,(50\,pF) = \boxed{280\,pF}$