Homework #3

(25-1) $\Delta U = q\Delta V = e(1.2\times10^9\,V) = \boxed{1.2\times10^9\,eV}$

(25-2) a) $W_{field} = -q\Delta V = -q(V_B - V_A)$

so $V_B - V_A = -\dfrac{W}{q} = \dfrac{-3.94\times10^{-19}J}{-1.6\times10^{-19}C} = \boxed{2.46\,V}$

(The voltage rises in going from $A \rightarrow B$)

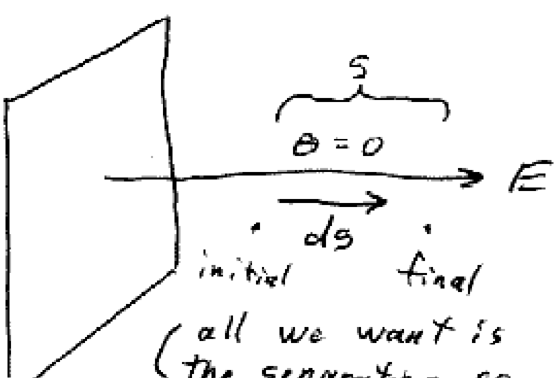
b) The voltage at points B and C are the same since the lie on an equipotential. So

$V_C - V_A = V_B - V_A = \boxed{2.46\,V}$

c) $\boxed{V_C - V_B = 0}$ because $V_C = V_B$

(25-3) The electric field due to an infinite conducting sheet, as we found in class, is

$$E = \frac{\sigma}{2\varepsilon_0}$$



(all we want is the separation, so which is which doesn't matter!)

Use $V_f - V_i = -\int E\,ds\cos\theta$

or $V_f - V_i = -Es$

In magnitude,

$|\Delta V| = |E|s$

Therefore $s = \dfrac{|\Delta V|}{E} = \dfrac{|\Delta V|}{\sigma/2\varepsilon_0} = \dfrac{|\Delta V|2\varepsilon_0}{\sigma}$

$= \dfrac{(50\,V)(2)(8.85\times10^{-12}\,\frac{C^2}{Nm^2})}{0.1\times10^{-6}\,C/m^2}$

$$\boxed{S = 8.85 \times 10^{-3}\ m}$$

(25-4) a) $\underset{B}{\cdot}$ $\quad d_2 = 1m \quad$ $\underset{q = 1\mu C}{\oplus}$ $\quad d_1 = 2m \quad$ $\underset{A}{\cdot}$

$$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d_1} - \frac{q}{d_2}\right) = k\left(\frac{q}{d_1} - \frac{q}{d_2}\right)$$

$$= kq\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$$

$$= \left(9\times10^{9}\,\frac{Nm^2}{C^2}\right)\left(10^{-6}C\right)\left(\frac{1}{2m} - \frac{1}{1m}\right)$$

$$= \boxed{-4500\,V}$$

b) It doesn't matter if B is now above the charge, since d_2 hasn't changed. Only distance counts for V due a point charge, so the answer is the same, $\boxed{V_A - V_B = -4500\ V}$

(25-5) $$V(P) = k\left(\frac{+5q}{d} + \frac{+5q}{d} + \frac{-5q}{d} + \frac{-5q}{2d}\right)$$

$$= \boxed{k\,\frac{5q}{2d}}$$

(25-6) Start with the expression for V on the axis of a charged disk, which we found in class

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$

In this problem, $z = 2R$. If the disk had no hole, the voltage at P would be

$$V_{\substack{complete\\disk}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{(2R)^2 + R^2} - 2R\right)$$

$$V_{\substack{\text{complete}\\ \text{disk}}} = \frac{\sigma}{2\varepsilon_0} R(\sqrt{5} - 2)$$

The disk in the problem can be thought of as a complete disk with a small central disk removed:

The voltage at P due to the small disk would be

$$V_{\substack{\text{small}\\ \text{disk}}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + r^2} - z\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{(2R)^2 + (0.2R)^2} - 2R\right)$$

$$= \frac{\sigma}{2\varepsilon_0} R\left(\sqrt{4.04} - 2\right)$$

Thus the voltage at P due to the disk with the hole is

$$V_{\substack{\text{disk}\\ \text{with hole}}} = V_{\substack{\text{complete}\\ \text{disk}}} - V_{\substack{\text{small}\\ \text{disk}}}$$

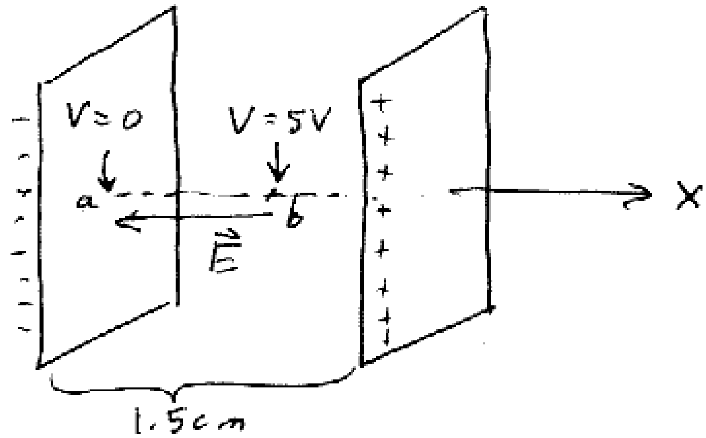
$$= \frac{\sigma}{2\varepsilon_0} R(\sqrt{5} - 2) - \frac{\sigma}{2\varepsilon_0} R(\sqrt{4.04} - 2)$$

$$= \frac{\sigma}{2\varepsilon_0} R(\sqrt{5} - \sqrt{4.04})$$

$$= \frac{\sigma}{2\varepsilon_0} R(0.226)$$

$$= \boxed{0.113 \frac{\sigma}{\varepsilon_0} R}$$

(25-7)



$$E_x = -\frac{\partial V}{\partial x}$$

$$= -\frac{(V_b - V_a)}{x_b - x_a}$$

$$= -\frac{(5V - 0V)}{0.015m/2}$$

$$\boxed{E_x = -667\ \frac{V}{m}}$$

The (–) sign means that $\vec{E}$ is in the –x direction.

(25-8) $V = (2.0\ \frac{V}{m^2})x^2 - (3.0\ \frac{V}{m^2})y^2$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$$

$$\vec{E} = -(2.0\frac{V}{m^2})(\frac{dx^2}{dx})\hat{i} - (-3.0\frac{V}{m^2})(\frac{dy^2}{dy})\hat{j}$$

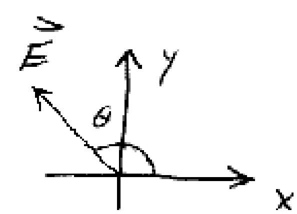
$$\vec{E} = -(2.0\frac{V}{m^2})(2x)\hat{i} - (-3.0\frac{V}{m^2})(2y)\hat{j}$$

$$\vec{E} = -4.0\frac{V}{m^2}x\hat{i} + 6.0\frac{V}{m^2}y\hat{j}$$ general formula for $\vec{E}$

At $x = 3m$, $y = 2m$

$$\vec{E} = -(4.0\frac{V}{m^2})(3m)\hat{i} + (6.0\frac{V}{m^2})(2m)\hat{j}$$

$$\boxed{\vec{E} = -12\frac{V}{m}\hat{i} + 12\frac{V}{m}\hat{j}}$$

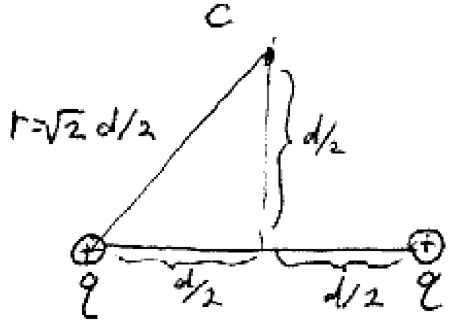


The magnitude of $\vec{E}$ is

$$E = \sqrt{(-12\frac{V}{m})^2 + (12\frac{V}{m})^2} = \boxed{17.0\ \frac{V}{m}}$$

$$\tan\theta = \frac{E_y}{E_x} = \frac{12\frac{V}{m}}{-12\frac{V}{m}} = -1 \Rightarrow \boxed{\theta = 135^\circ \text{ ccw from +x axis}}$$

(25-9)



a) $$V_c = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{q}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{2q}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{2q}{\sqrt{2}d/2}$$

$$V_c = \frac{1}{4\pi\varepsilon_0}\,\frac{2\sqrt{2}\,q}{d}$$

Using $q = 2\mu C$ and $d = 2$ cm and $\frac{1}{4\pi\varepsilon_0} = k$,

$$V_c = \left(8.99\times10^{9}\ \frac{Nm^2}{C^2}\right)\frac{2\sqrt{2}\,(2\times10^{-6}C)}{0,02\ m}$$

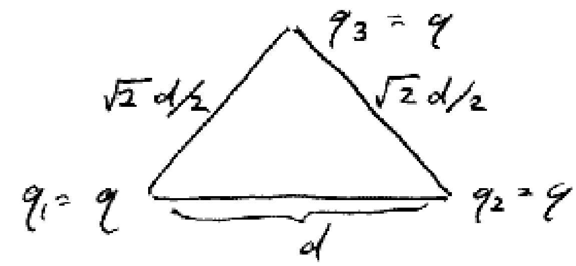
$$= \boxed{2,54\times10^{6}\ V}$$

b) $$W = q(\Delta V) = q(V_f - V_i) = q(V_c - V_\infty)$$

so $$W = (2\times10^{-6}C)(2,54\times10^{6}\ V - 0)$$

$$= \boxed{5,08\ J}$$

c) $$U = \sum_{\substack{\text{all}\\ \text{pairs}}} \frac{1}{4\pi\varepsilon_0}\,\frac{q_i q_j}{r_{ij}}$$



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{d} + \frac{q^2}{\sqrt{2}d/2} + \frac{q^2}{\sqrt{2}d/2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\,q^2\left(\frac{1}{d} + \frac{\sqrt{2}}{d} + \frac{\sqrt{2}}{d}\right)$$

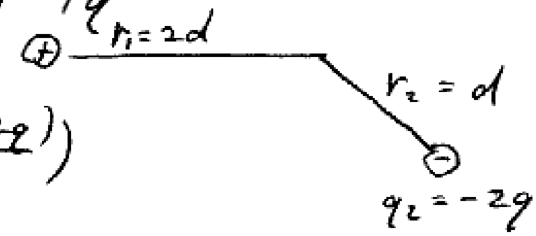
$$= \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{d}\left(1 + 2\sqrt{2}\right)$$

$$U = \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(2\times10^{-6}C)^2}{0.02m}\,(1+2\sqrt{2})$$

$$= \boxed{6.88\ J}$$

(25-10) Use $W = q(\Delta V) = q(V_f - V_i)$ with $V_i = 0V$ at infinity!

$$V_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

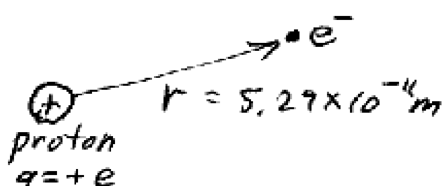


$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q}{2d}+\frac{(-2q)}{d}\right)$$

$$= 0V$$

So $W = q(V_f - V_i) = 5q(0V - 0V) = \boxed{0}$

(25-11) a)



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$= \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(1.6\times10^{-19}C)}{5.29\times10^{-11}m}$$

$$= \boxed{27.2\ V}$$

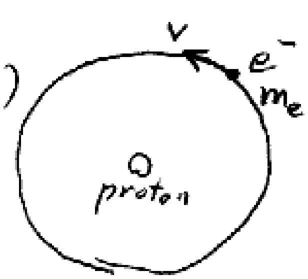
b) $U = k\,\frac{q_p\,q_e}{r} = k\,\frac{(e)(-e)}{r}$

$$= -\left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(1.6\times10^{-19}C)^2}{5.29\times10^{-11}m}$$

$$= -4.35\times10^{-18}J\left(\frac{1\,eV}{1.6\times10^{-19}J}\right)$$

$$= \boxed{-27.2\,eV}$$

c)



$$F = m_e a_c$$

$$k\,\frac{|q_p||q_e|}{r^2} = m_e\frac{v^2}{r}$$

We want the electron's kinetic energy, $K = \frac{1}{2} m_e v^2$. From the last equation on the previous page,

$$K = \frac{1}{2} m_e v^2 = \frac{1}{2} k \frac{|q_1||q_2|}{r}$$

or

$$K = \frac{1}{2}\left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(1.6\times10^{-19}\,C)^2}{5.29\times10^{-11}\,m}$$

$$= 2.175\times10^{-18}\,J\left(\frac{1\,eV}{1.6\times10^{-19}\,J}\right)$$

$$= \boxed{13.6\,eV}$$

d) The work equals the change in the total energy of the electron. Assuming that the electron is at rest at infinity,

$$W = E_f - E_i$$

$$= (K_f + U_f) - (K_i + U_i)$$

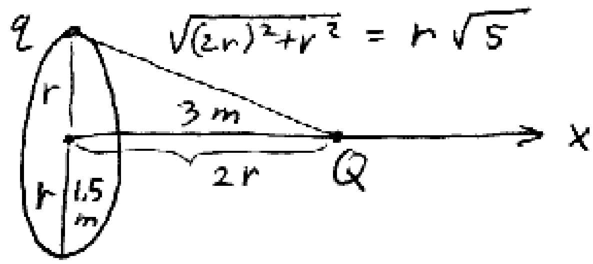
$$= (0 + 0) - (K_i + U_i)$$

$$= -(13.6\,eV - 27.2\,eV)$$

$$= \boxed{13.6\,eV}$$

The ionization energy of hydrogen (in the ground state)

(25-12)



$r = 1.5\,m$, $q = -9\,nC$
$Q = -6\,pC$

At $x = 0$, $V_f = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (only distance counts!)

At $x = 3m$, $V_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r\sqrt{5}}$

So $W = Q(\Delta V) = Q(V_f - V_i)$

$$= (-6\times10^{-12}C)\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r\sqrt{5}}\right)$$

$$= (-6\times10^{-12}C)\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{r}\right)\left(1 - \frac{1}{\sqrt{5}}\right)$$

$$= (-6\times10^{-12}C)\left(8.99\times10^{9}\frac{Nm^2}{C^2}\right)\left(\frac{-9\times10^{-9}C}{1.5m}\right)\left(1-\frac{1}{\sqrt{5}}\right)$$

$$= \boxed{1.79\times10^{-10}J}$$