

Homework #3

$$(25-1) \quad \Delta U = q \Delta V = e (1.2 \times 10^9 \text{ V}) = \boxed{1.2 \times 10^9 \text{ eV}}$$

$$(25-2) \quad a) \quad W_{\text{field}} = -q \Delta V = -q (V_B - V_A)$$

$$\text{so } V_B - V_A = \frac{-W}{q} = \frac{-3.94 \times 10^{-19} \text{ J}}{-1.6 \times 10^{-19} \text{ C}} = \boxed{2.46 \text{ V}}$$

(The voltage rises in going from A \rightarrow B)

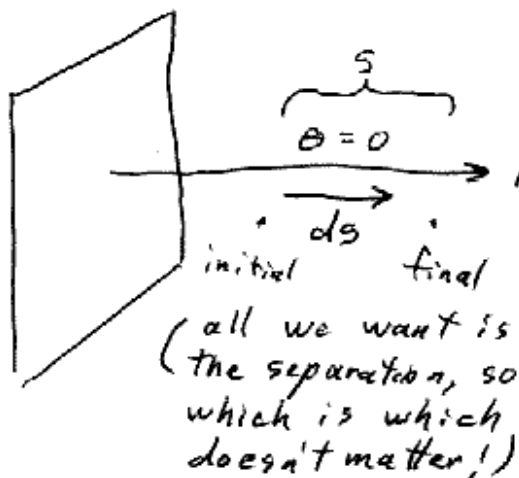
b) The voltage at points B and C are the same since they lie on an equipotential. So

$$V_C - V_A = V_B - V_A = \boxed{2.46 \text{ V}}$$

$$c) \quad \boxed{V_C - V_B = 0} \quad \text{because } V_C = V_B$$

(25-3) The electric field due to an infinite conducting sheet, as we found in class, is

$$E = \frac{\sigma}{2\epsilon_0}$$



$$\text{Use } V_f - V_i = - \int E ds \cos \theta$$

$$\text{or } V_f - V_i = - E s$$

In magnitude,

$$|\Delta V| = |E| s$$

$$\text{Therefore } s = \frac{|\Delta V|}{E} = \frac{|\Delta V|}{\sigma / 2\epsilon_0} = \frac{|\Delta V| 2\epsilon_0}{\sigma}$$

$$= \frac{(50 \text{ V})(2)(8.85 \times 10^{-12} \text{ N m}^2/\text{C}^2)}{0.1 \times 10^{-6} \text{ C/m}^2}$$

$$S = 8.85 \times 10^{-3} \text{ m}$$

(25-4) a) \cdot d_2 \oplus d_1 \cdot
 $B = 1\text{m}$ q $= 2\text{m}$ A
 $= 1\mu\text{C}$

$$V_A - V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{d_1} - \frac{q}{d_2} \right) = k \left(\frac{q}{d_1} - \frac{q}{d_2} \right)$$

$$= kq \left(\frac{1}{d_1} - \frac{1}{d_2} \right)$$

$$= (9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}) (10^{-6} \text{C}) \left(\frac{1}{2\text{m}} - \frac{1}{1\text{m}} \right)$$

$$= \boxed{-4500 \text{ V}}$$

b) It doesn't matter if B is now above the charge, since d_2 hasn't changed. Only distance counts for V due a point charge, so the answer is the same,

$$\boxed{V_A - V_B = -4500 \text{ V}}$$

(25-5) $V(P) = k \left(\frac{+5q}{d} + \frac{+5q}{d} + \frac{-5q}{d} + \frac{-5q}{2d} \right)$

$$= \boxed{k \frac{5q}{2d}}$$

(25-6) Start with the expression for V on the axis of a charged disk, which we found in class

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z)$$

In this problem, $z = 2R$. If the disk had no hole, the voltage at P would be

$$V_{\text{complete disk}} = \frac{\sigma}{2\epsilon_0} (\sqrt{(2R)^2 + R^2} - 2R)$$

$$V_{\text{complete disk}} = \frac{\sigma}{2\epsilon_0} R(\sqrt{5} - 2)$$

The disk in the problem can be thought of as a complete disk with a small central disk removed:



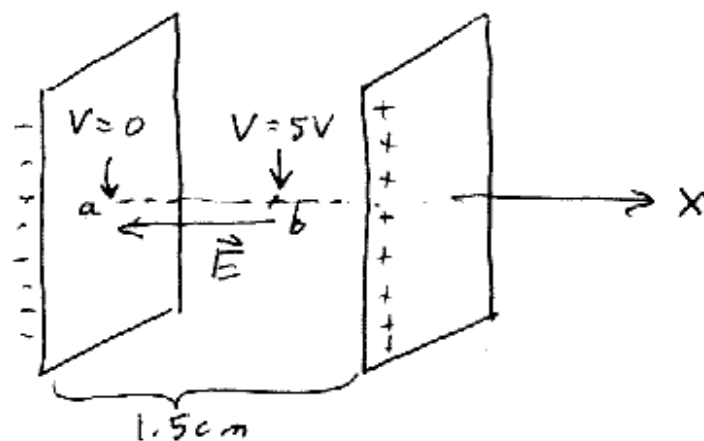
The voltage at P due to the small disk would be

$$\begin{aligned} V_{\text{small disk}} &= \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + r^2} - z) \\ &= \frac{\sigma}{2\epsilon_0} (\sqrt{(2R)^2 + (0.2R)^2} - 2R) \\ &= \frac{\sigma}{2\epsilon_0} R(\sqrt{4.04} - 2) \end{aligned}$$

Thus the voltage at P due to the disk with the hole is

$$\begin{aligned} V_{\text{disk with hole}} &= V_{\text{complete disk}} - V_{\text{small disk}} \\ &= \frac{\sigma}{2\epsilon_0} R(\sqrt{5} - 2) - \frac{\sigma}{2\epsilon_0} R(\sqrt{4.04} - 2) \\ &= \frac{\sigma}{2\epsilon_0} R(\sqrt{5} - \sqrt{4.04}) \\ &= \frac{\sigma}{2\epsilon_0} R(0.226) \\ &= \boxed{0.113 \frac{\sigma}{\epsilon_0} R} \end{aligned}$$

(25-7)



$$\begin{aligned}
 E_x &= -\frac{\partial V}{\partial x} \\
 &= -\frac{(V_b - V_a)}{x_b - x_a} \\
 &= -\frac{(5\text{ V} - 0\text{ V})}{0.015\text{ m}/2}
 \end{aligned}$$

$$E_x = -667 \frac{\text{V}}{\text{m}}$$

The (-) sign means that \vec{E} is in the $-x$ direction.

(25-8)

$$V = (2.0 \frac{\text{V}}{\text{m}^2})x^2 - (3.0 \frac{\text{V}}{\text{m}^2})y^2$$

$$\vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j}$$

$$\vec{E} = -(2.0 \frac{\text{V}}{\text{m}^2})\left(\frac{dx^2}{dx}\right) \hat{i} - (-3.0 \frac{\text{V}}{\text{m}^2})\left(\frac{dy^2}{dy}\right) \hat{j}$$

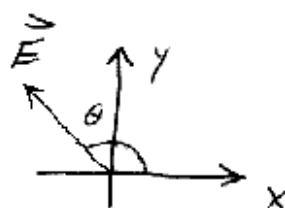
$$\vec{E} = -(2.0 \frac{\text{V}}{\text{m}^2})(2x) \hat{i} - (-3.0 \frac{\text{V}}{\text{m}^2})(2y) \hat{j}$$

$$\vec{E} = -4.0 \frac{\text{V}}{\text{m}^2} x \hat{i} + 6.0 \frac{\text{V}}{\text{m}^2} y \hat{j} \quad \text{general formula for } \vec{E}$$

At $x = 3\text{ m}$, $y = 2\text{ m}$

$$\vec{E} = -(4.0 \frac{\text{V}}{\text{m}^2})(3\text{ m}) \hat{i} + (6.0 \frac{\text{V}}{\text{m}^2})(2\text{ m}) \hat{j}$$

$$\vec{E} = -12 \frac{\text{V}}{\text{m}} \hat{i} + 12 \frac{\text{V}}{\text{m}} \hat{j}$$



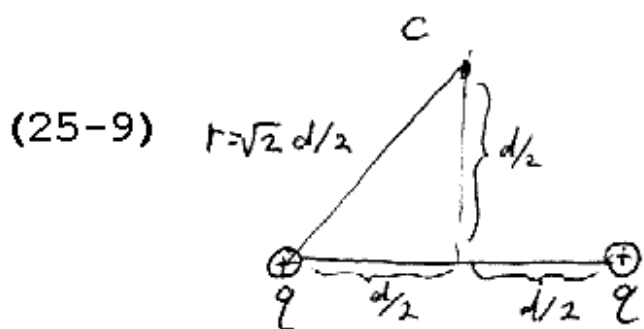
The magnitude of \vec{E} is

$$E = \sqrt{(-12 \frac{\text{V}}{\text{m}})^2 + (12 \frac{\text{V}}{\text{m}})^2} = 17.0 \frac{\text{V}}{\text{m}}$$

$$\tan \theta = \frac{E_y}{E_x} = \frac{12 \frac{\text{V}}{\text{m}}}{-12 \frac{\text{V}}{\text{m}}} = -1 \Rightarrow$$

$$\theta = 135^\circ$$

ccw from
+x axis



$$\begin{aligned}
 a) \quad V_c &= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} + \frac{q}{r} \right) \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2q}{r} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2q}{\sqrt{2}d/2} \\
 V_c &= \frac{1}{4\pi\epsilon_0} \frac{2\sqrt{2}q}{d}
 \end{aligned}$$

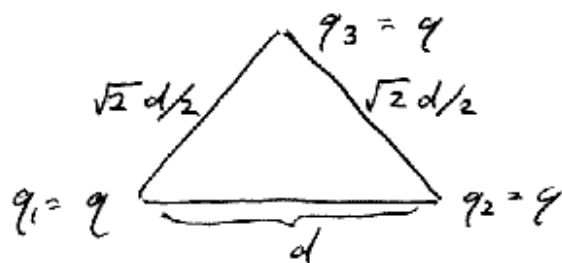
Using $q = 2\mu\text{C}$ and $d = 2\text{ cm}$ and $\frac{1}{4\pi\epsilon_0} = k$,

$$\begin{aligned}
 V_c &= \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{2\sqrt{2} (2 \times 10^{-6} \text{C})}{0.02 \text{ m}} \\
 &= \boxed{2.54 \times 10^6 \text{ V}}
 \end{aligned}$$

$$b) \quad W = q(\Delta V) = q(V_f - V_i) = q(V_c - V_\infty)$$

$$\begin{aligned}
 \text{so } W &= (2 \times 10^{-6} \text{ C}) (2.54 \times 10^6 \text{ V} - 0) \\
 &= \boxed{5.08 \text{ J}}
 \end{aligned}$$

$$c) \quad U = \sum_{\text{all pairs}} \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}}$$



$$\begin{aligned}
 U &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \\
 &= \frac{1}{4\pi\epsilon_0} \left(\frac{q^2}{d} + \frac{q^2}{\sqrt{2}d/2} + \frac{q^2}{\sqrt{2}d/2} \right) \\
 &= \frac{1}{4\pi\epsilon_0} q^2 \left(\frac{1}{d} + \frac{\sqrt{2}}{d} + \frac{\sqrt{2}}{d} \right) \\
 &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} (1 + 2\sqrt{2})
 \end{aligned}$$

$$U = \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{(2 \times 10^{-6} \text{C})^2}{0.02 \text{m}} (1 + 2\sqrt{2})$$

$$= \boxed{6.88 \text{ J}}$$

(25-10) Use $W = q(\Delta V) = q(V_f - V_i)$ with $V_i = 0 \text{ V}$ at infinity.

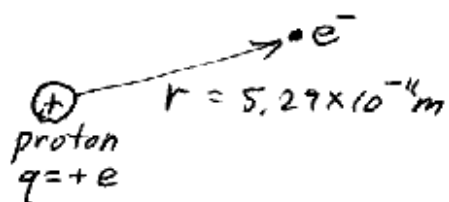
$$V_f = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \quad \begin{array}{l} q_1 = 4q \\ r_1 = 2d \\ q_2 = -2q \\ r_2 = d \end{array}$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{4q}{2d} + \frac{(-2q)}{d} \right)$$

$$= 0 \text{ V}$$

So $W = q(V_f - V_i) = 5q(0 \text{ V} - 0 \text{ V}) = \boxed{0}$

(25-11) a)



$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$= \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{(1.6 \times 10^{-19} \text{C})}{5.29 \times 10^{-11} \text{m}}$$

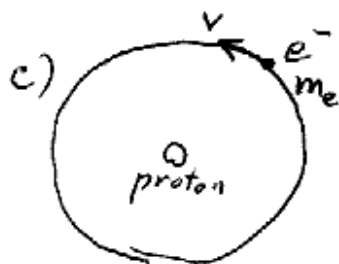
$$= \boxed{27.2 \text{ V}}$$

b) $U = k \frac{q_1 q_2}{r} = k \frac{(e)(-e)}{r}$

$$= \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{(1.6 \times 10^{-19} \text{C})^2}{5.29 \times 10^{-11} \text{m}}$$

$$= -4.35 \times 10^{-18} \text{ J} \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right)$$

$$= \boxed{-27.2 \text{ eV}}$$



$$F = m_e a_c$$

$$k \frac{|q_1 q_2|}{r^2} = m_e \frac{v^2}{r}$$

We want the electron's kinetic energy,
 $K = \frac{1}{2} m_e v^2$. From the last equation on
 the previous page,

$$K = \frac{1}{2} m_e v^2 = \frac{1}{2} k \frac{|q_1 q_2|}{r}$$

or

$$K = \frac{1}{2} (8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}) \frac{(1.6 \times 10^{-19} \text{C})^2}{5.29 \times 10^{-11} \text{m}}$$

$$= 2.175 \times 10^{-18} \text{J} \left(\frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}} \right)$$

$$= \boxed{13.6 \text{eV}}$$

d) The work equals the change in the total energy of the electron. Assuming that the electron is at rest at infinity,

$$W = E_f - E_i$$

$$= (K_f + U_f) - (K_i + U_i)$$

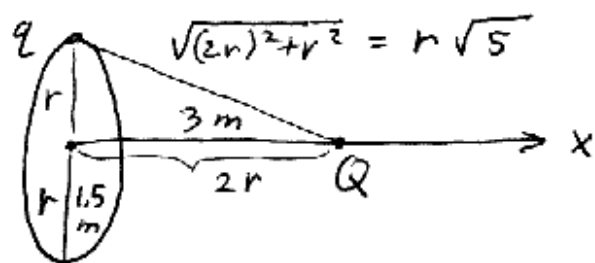
$$= (0 + 0) - (K_i + U_i)$$

$$= -(13.6 \text{eV} - 27.2 \text{eV})$$

$$= \boxed{13.6 \text{eV}}$$

The ionization energy of hydrogen (in the ground state)

(25-12)



$$r = 1.5 \text{ m}, \quad q = -9 \text{ nC}$$

$$Q = -6 \text{ pC}$$

$$\text{At } x = 0, \quad V_f = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (\text{only distance counts!})$$

$$\text{At } x = 3 \text{ m}, \quad V_i = \frac{1}{4\pi\epsilon_0} \frac{q}{r\sqrt{5}}$$

$$\text{So } W = Q(\Delta V) = Q(V_f - V_i)$$

$$= (-6 \times 10^{-12} \text{ C}) \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r} - \frac{1}{4\pi\epsilon_0} \frac{q}{r\sqrt{5}} \right)$$

$$= (-6 \times 10^{-12} \text{ C}) \left(\frac{1}{4\pi\epsilon_0} \right) \left(\frac{q}{r} \right) \left(1 - \frac{1}{\sqrt{5}} \right)$$

$$= (-6 \times 10^{-12} \text{ C}) \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \left(\frac{-9 \times 10^{-9} \text{ C}}{1.5 \text{ m}} \right) \left(1 - \frac{1}{\sqrt{5}} \right)$$

$$= \boxed{1.79 \times 10^{-10} \text{ J}}$$