Homework #11

(38-1) $\text{Speed} = \frac{6\,ly}{6y + 2y} = \frac{3}{4}\,\frac{ly}{y}$

Since the speed of light is $c = 1\,\frac{ly}{y}$,

$$\boxed{\text{speed} = \frac{3}{4}c}$$

(38-2) $\Delta t_{rest} = 2.2\,\mu s$ and $\Delta t_{moving} = 16\,\mu s$

Use $\Delta t_{moving} = \dfrac{\Delta t_{rest}}{\sqrt{1 - v^2/c^2}}$

$$\sqrt{1 - v^2/c^2} = \frac{\Delta t_{rest}}{\Delta t_{moving}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{\Delta t_{rest}}{\Delta t_{moving}}\right)^2$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{\Delta t_{rest}}{\Delta t_{moving}}\right)^2}$$

$$= \sqrt{1 - \left(\frac{2.2\,\mu s}{16\,\mu s}\right)^2}$$

$$= \boxed{0.991}$$

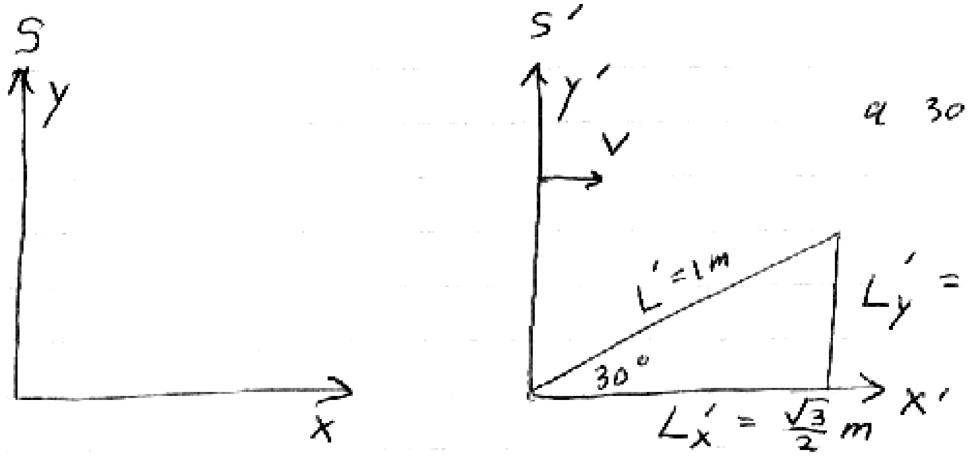
(38-3) a) At the end of the one-way trip, you will be six month older and the Earth will be 500 years older. Using this equation with $\Delta t_{rest} = 0.5y$ and $\Delta t_{moving} = 500y$, we get

$$\frac{v}{c} = \sqrt{1 - \left(\frac{\Delta t_{rest}}{\Delta t_{moving}}\right)^2}$$

$$= \sqrt{1 - \left(\frac{0.5y}{500y}\right)^2}$$

$$= \boxed{0.9999995}$$

(38-4)



a 30° - 60° - 90° triangle

As measured from frame S, the y-length is unchanged, since it is perpendicular to the motion

$$L_y = \frac{1}{2} m$$

The x-length is length contracted

$$L_x = L_x' \sqrt{1 - v^2/c^2}$$

$$= \frac{\sqrt{3}}{2} m \sqrt{1 - \left(\frac{0.9c}{c}\right)^2}$$

$$= 0.377 m$$

So the length of the stick as measured in frame S is

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{(0.5m)^2 + (0.372m)^2}$$

$$\boxed{L = 0.626 m}$$

(38-5) a) Earth sees a moving spaceship, and measures a time for the trip

$$\Delta t_{moving} = \frac{\text{distance}}{\text{speed}} = \frac{26\ ly}{0.99\ ly/y} = \boxed{26.3\ y}$$

b) Earth will have to wait another 26 years for a signal to travel from Vega to Earth, so

$$26.3\ y + 26\ y = 52.3\ y$$

will have elapsed

c) We can calculate that the pilot has aged

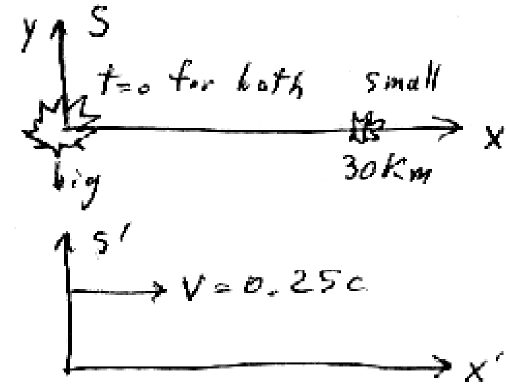
$$\Delta t_{rest} = \Delta t_{moving}\sqrt{1 - v^2/c^2}$$
$$= 26.3\ y\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}$$
$$= \boxed{3.70\ y}$$

(38-6)

$$X' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
$$= \frac{10^5\ m - (0.95)(3\times10^8\ m/sec)(200\times10^{-6}\ sec)}{\sqrt{1 - (0.95c/c)^2}}$$
$$= 1.38\times10^5\ m = \boxed{138\ Km}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$
$$= \frac{200\times10^{-6}\ sec - (0.95)(3\times10^8\ m/sec)(10^5\ m)/(3\times10^8\ \frac{m}{sec})^2}{\sqrt{1 - (0.95c/c)^2}}$$
$$= -3.74\times10^{-4}\ sec = \boxed{-374\ \mu sec}$$

(38-7) Let the experimenter be in frame S, and let's say flashes both occurred at $t=0$. The space coordinates of the two flashes are (as measured in frame S)



$$x_{big} = 0 \qquad t_{big} = 0$$

$$x_{small} = 30.0\,km \quad t_{small} = 0$$

As measured from frame S',

$$t'_{big} = \frac{t_{big} - V x_{big}/c^2}{\sqrt{1 - V^2/c^2}}$$

$$= \frac{0 - (0.25c)(0)/c^2}{\sqrt{1 - (0.25c/c)^2}}$$

$$\boxed{t'_{big} = 0}$$

$$t'_{small} = \frac{t_{small} - V x_{small}/c^2}{\sqrt{1 - V^2/c^2}}$$

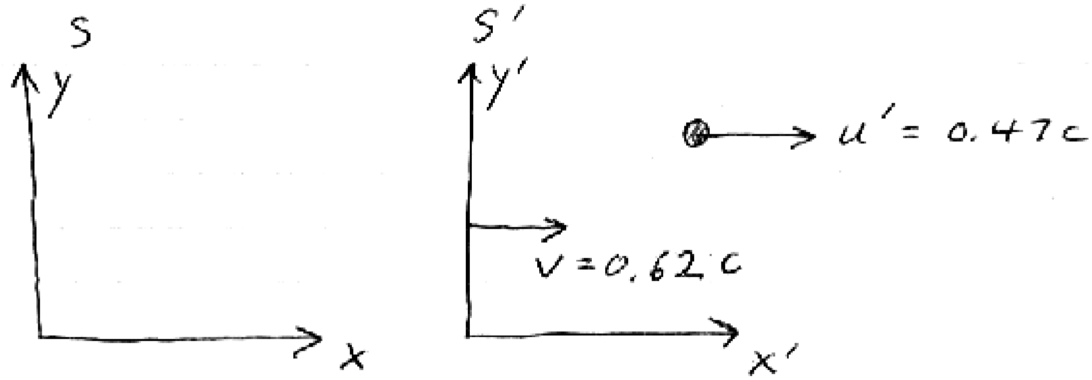
$$= \frac{0 - (0.25c)(3\times10^4\,m)/c\,(3\times10^8\,m/sec)}{\sqrt{1 - (0.25c/c)^2}}$$

$$\boxed{t'_{small} = -2.58\times10^{-5}\,sec}$$

The time interval is thus $\boxed{25.8\,\mu sec}$

b) The small flash occurred first, as measured in frame S'.

(38-8) a)



$$u = \frac{u' + V}{1 + u'V/c^2} = \frac{0.47c + 0.62c}{1 + \frac{(0.47c)(0.62c)}{c^2}} = \boxed{0.844c}$$

Classically, we would expect $u = u' + V = \boxed{1.09c}$

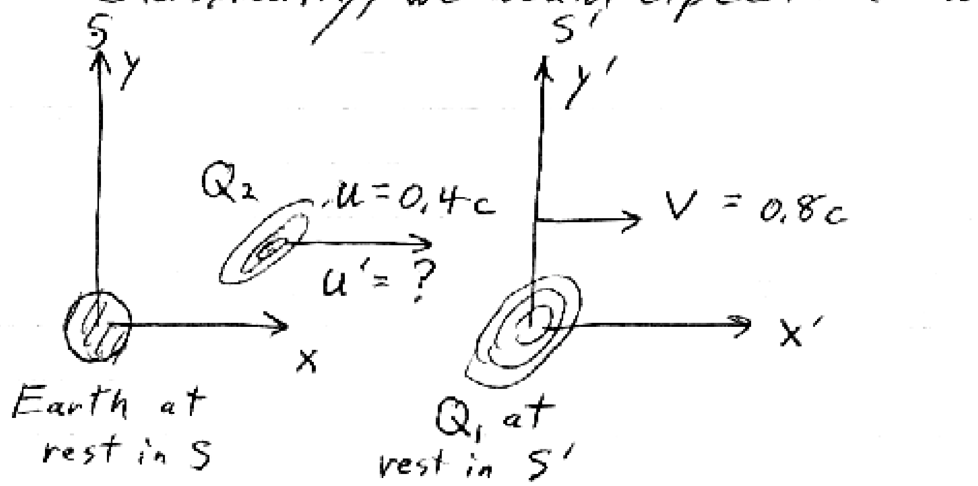
b) If the particle reverses its direction, so $u' = -0.47c$, we then have

$$u = \frac{u' + V}{1 + u'V/c^2} = \frac{-0.47c + 0.62c}{1 + \frac{(-0.47c)(0.62c)}{c^2}}$$

$$= \boxed{0.212c}$$

Classically, we would expect $u = u' + V = \boxed{0.15c}$

(38-9)



u' is the speed of quasar Q_2 as measured by quasar Q_1

$$u' = \frac{u - V}{1 - uV/c^2} = \frac{0.4c - 0.8c}{1 - \frac{(0.4c)(0.8c)}{c^2}} = \boxed{-0.588c}$$

away from Q_1

(38-10) a) For a moving source and stationary, the classic Doppler shift equation (eqn 18-52) is

$$f' = f\,\frac{V}{V+V_s} \qquad \text{(source moving away)}$$

Applying this to light, $f' = \frac{c}{\lambda'}$ and $f = \frac{c}{\lambda}$ and $V = c$

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\,\frac{c}{c+V_s}$$

$$\frac{\lambda'}{\lambda} = \frac{c+V_s}{c} = 1 + \frac{V_s}{c}$$

Here, $\frac{\lambda'}{\lambda} = 3$, so

$$3 = 1 + \frac{V_s}{c}$$

$$\boxed{\frac{V_s}{c} = 2}$$ twice the speed of light (wrong!)

b) Actually, $f = f_o\sqrt{\frac{1-\beta}{1+\beta}}$

$$\frac{c}{\lambda} = \frac{c}{\lambda_o}\sqrt{\frac{1-\beta}{1+\beta}}$$

$$\frac{\lambda}{\lambda_o} = \sqrt{\frac{1+\beta}{1-\beta}}$$

Again, $\frac{\lambda}{\lambda_o} = 3$, so squaring gives

$$(3)^2 = 9 = \frac{1+\beta}{1-\beta}$$

$$9(1-\beta) = 1+\beta$$

$$10\beta = 8 \Rightarrow \boxed{\beta = \frac{v}{c} = 0.8}$$

(38-11) a) The rest energy of a proton is

$$m_p c^2 = 938 \text{ MeV}$$

The Lorentz factor for $v = 0.99c$ is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 7.09$$

The proton's total energy is ($1 \text{ GeV} = 10^3 \text{ MeV}$)

$$E = \gamma m_p c^2 = (7.09)(938 \text{ MeV}) = \boxed{6.65 \text{ GeV}}$$

Its kinetic energy is

$$K = E - m_p c^2 = 6.65 \text{ GeV} - 0.938 \text{ GeV}$$

$$K = \boxed{5.71 \text{ GeV}}$$

Its momentum comes from

$$E^2 = (pc)^2 + (m_p c^2)^2$$

or

$$pc = \sqrt{E^2 - (m_p c^2)^2}$$

$$= \sqrt{(6.65 \text{ GeV})^2 - (0.938 \text{ GeV})^2}$$

$$pc = 6.58 \text{ GeV}$$

Thus $\boxed{p = 6.58 \frac{\text{GeV}}{c}}$

b) The electron's rest energy is $m_e c^2 = 0.511\,MeV$

So $E = \gamma m_e c^2 = (7.09)(0.511\,MeV) = \boxed{3.62\,MeV}$

$K = E - m_e c^2 = 3.62\,MeV - 0.511\,MeV$

$K = \boxed{3.11\,MeV}$

$pc = \sqrt{E^2 - (m_e c^2)^2}$

$= \sqrt{(3.62\,MeV)^2 - (0.511\,MeV)^2}$

$= 3.58\,MeV$

Thus $\boxed{p = 3.58\,\frac{MeV}{c}}$

(38-12) The rest energy of the asprin tablet is

$E = mc^2 = (320 \times 10^{-6}\,kg)(3 \times 10^8\,\frac{m}{sec})^2$

$= 2.88 \times 10^{13}\,J$

This is equivalent to

$\frac{2.88 \times 10^{13}\,J}{1.3 \times 10^8\,J/gal} = 2.22 \times 10^5$ gal of gasoline

At $30\,\frac{mi}{gal}$, a car could drive

$30\,\frac{mi}{gal}(2.22 \times 10^5\,gal) = \boxed{6.66 \times 10^6 \text{ miles !}}$