

Homework #11

$$(38-1) \text{ speed} = \frac{6 \text{ ly}}{6y + 2y} = \frac{3}{4} \frac{\text{ly}}{y}$$

Since the speed of light is $c = 1 \frac{\text{ly}}{y}$,

$$\boxed{\text{speed} = \frac{3}{4} c}$$

$$(38-2) \Delta t_{\text{rest}} = 2.2 \mu\text{s} \text{ and } \Delta t_{\text{moving}} = 16 \mu\text{s}$$

$$\text{Use } \Delta t_{\text{moving}} = \frac{\Delta t_{\text{rest}}}{\sqrt{1 - v^2/c^2}}$$

$$\sqrt{1 - v^2/c^2} = \frac{\Delta t_{\text{rest}}}{\Delta t_{\text{moving}}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{\Delta t_{\text{rest}}}{\Delta t_{\text{moving}}} \right)^2$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{\Delta t_{\text{rest}}}{\Delta t_{\text{moving}}} \right)^2}$$

$$= \sqrt{1 - \left(\frac{2.2 \mu\text{s}}{16 \mu\text{s}} \right)^2}$$

$$= \boxed{0.991}$$

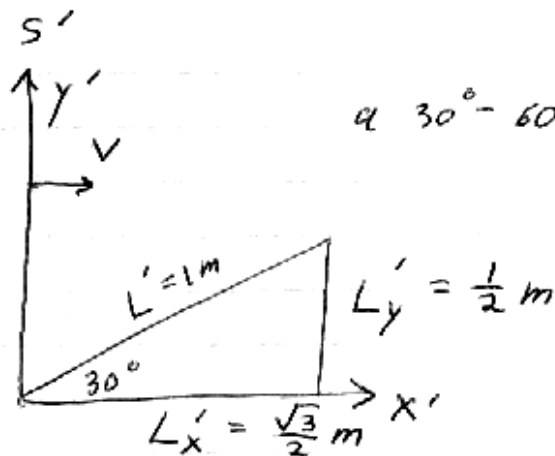
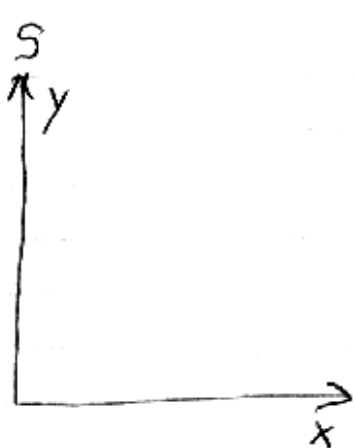
(38-3) a) At the end of the one-way trip, you will be six months older and the Earth will be 500 years older. Using this equation with $\Delta t_{\text{rest}} = 0.5y$ and $\Delta t_{\text{moving}} = 500y$, we get

$$\frac{v}{c} = \sqrt{1 - \left(\frac{\Delta t_{\text{rest}}}{\Delta t_{\text{moving}}} \right)^2}$$

$$= \sqrt{1 - \left(\frac{0.5y}{500y} \right)^2}$$

$$= \boxed{0.9999995}$$

(38-4)

a $30^\circ - 60^\circ - 90^\circ$ triangle

As measured from frame S, the y-length is unchanged, since it is perpendicular to the motion

$$L_y = \frac{1}{2}\text{ m}$$

The x-length is length contracted

$$\begin{aligned} L_x &= L'_x \sqrt{1 - \frac{v^2}{c^2}} \\ &= \frac{\sqrt{3}}{2}\text{ m} \sqrt{1 - \left(\frac{0.9c}{c}\right)^2} \\ &= 0.377\text{ m} \end{aligned}$$

So the length of the stick as measured in frame S is

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{(0.377\text{ m})^2 + (0.5\text{ m})^2}$$

$$\boxed{L = 0.626\text{ m}}$$

- (38-5) a) Earth sees a moving spaceship, and measures a time for the trip

$$\Delta t_{\text{moving}} = \frac{\text{distance}}{\text{speed}} = \frac{26 \text{ ly}}{0.99 \text{ ly/y}} = \boxed{26.3 \text{ y}}$$

- b) Earth will have to wait another 26 years for a signal to travel from Vega to Earth, so

$$26.3 \text{ y} + 26 \text{ y} = 52.3 \text{ y}$$

will have elapsed

- c) We can calculate that the pilot has aged

$$\begin{aligned} \Delta t_{\text{rest}} &= \Delta t_{\text{moving}} \sqrt{1 - v^2/c^2} \\ &= 26.3 \text{ y} \sqrt{1 - \left(\frac{0.99c}{c}\right)^2} \\ &= \boxed{3.70 \text{ y}} \end{aligned}$$

(38-6)
$$X' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{10^5 \text{ m} - (0.95)(3 \times 10^8 \text{ m/sec})(200 \times 10^{-6} \text{ sec})}{\sqrt{1 - (0.95c)^2}}$$

$$= 1.38 \times 10^5 \text{ m} = \boxed{138 \text{ km}}$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

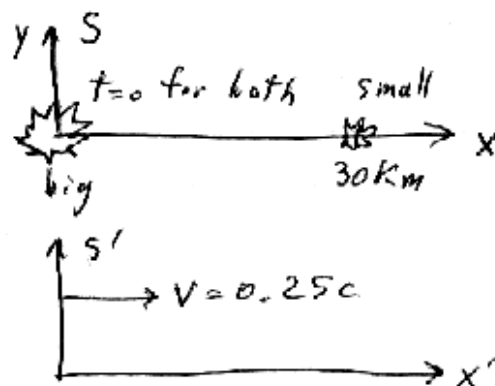
$$= \frac{200 \times 10^{-6} \text{ sec} - (0.95)(3 \times 10^8 \text{ m/sec})(10^5 \text{ m}) / (3 \times 10^8 \frac{\text{m}}{\text{sec}})^2}{\sqrt{1 - (0.95c)^2}}$$

$$= -3.74 \times 10^{-4} \text{ sec} = \boxed{-374 \mu \text{ sec}}$$

(38-7) Let the experimenter be in frame S , and let's say flashes both occurred at $t=0$. The space coordinates of the two flashes are (as measured in frame S)

$$x_{\text{big}} = 0 \quad t_{\text{big}} = 0$$

$$x_{\text{small}} = 30.0 \text{ km} \quad t_{\text{small}} = 0$$



As measured from frame S' ,

$$t'_{\text{big}} = \frac{t_{\text{big}} - vx_{\text{big}}/c^2}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{0 - (0.25c)(0)/c^2}{\sqrt{1 - (0.25c/c)^2}}$$

$$\boxed{t'_{\text{big}} = 0}$$

$$t'_{\text{small}} = \frac{t_{\text{small}} - vx_{\text{small}}/c^2}{\sqrt{1 - v^2/c^2}}$$

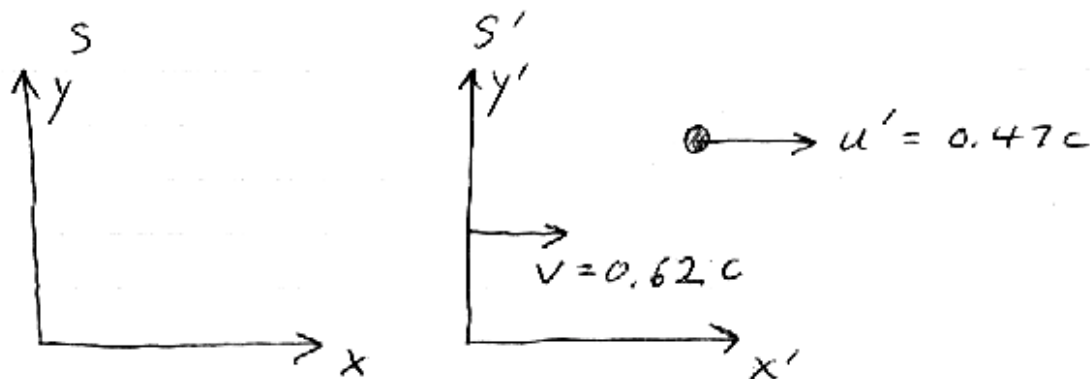
$$= \frac{0 - (0.25c)(3 \times 10^4 \text{ m})/c(3 \times 10^8 \text{ m/sec})}{\sqrt{1 - (0.25c/c)^2}}$$

$$\boxed{t'_{\text{small}} = -2.98 \times 10^{-5} \text{ sec}}$$

The time interval is thus $\boxed{25.8 \mu\text{sec}}$

b) The small flash occurred first, as measured in frame S' .

(38-8) a)



$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.47c + 0.62c}{1 + \frac{(0.47c)(0.62c)}{c^2}} = \boxed{0.844c}$$

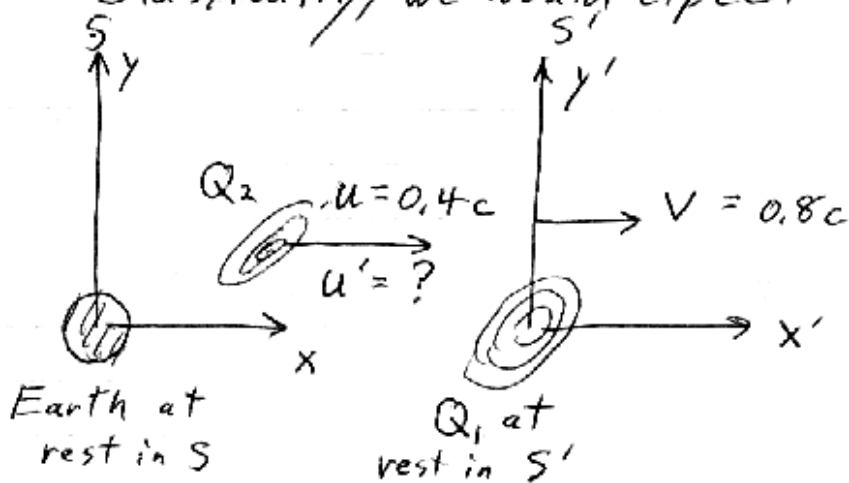
Classically, we would expect $u = u' + v = \boxed{1.09c}$

b) If the particle reverses its direction, so $u' = -0.47c$, we then have

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{-0.47c + 0.62c}{1 + \frac{(-0.47c)(0.62c)}{c^2}} = \boxed{0.212c}$$

Classically, we would expect $u = u' + v = \boxed{0.15c}$

(38-9)



u' is the speed of quasar Q_2 as measured by quasar Q_1

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.4c - 0.8c}{1 - \frac{(0.4c)(0.8c)}{c^2}} = \boxed{-0.588c}$$

away from Q_1

(38-10) a) For a moving source and stationary, the classic Doppler shift equation (eqn 18-52) is

$$f' = f \frac{v}{v + v_s} \quad (\text{source moving away})$$

Applying this to light, $f' = \frac{c}{\lambda'}$ and $f = \frac{c}{\lambda}$
and $v = c$

$$\frac{c}{\lambda'} = \frac{c}{\lambda} \frac{c}{c + v_s}$$

$$\frac{\lambda'}{\lambda} = \frac{c + v_s}{c} = 1 + \frac{v_s}{c}$$

Here, $\frac{\lambda'}{\lambda} = 3$, so

$$3 = 1 + \frac{v_s}{c}$$

$$\boxed{\frac{v_s}{c} = 2} \quad \text{twice the speed of light (wrong!)}$$

b) Actually, $f = f_0 \sqrt{\frac{1-\beta}{1+\beta}}$

$$\frac{c}{\lambda} = \frac{c}{\lambda_0} \sqrt{\frac{1-\beta}{1+\beta}}$$

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}$$

Again, $\frac{\lambda}{\lambda_0} = 3$, so squaring gives

$$(3)^2 = 9 = \frac{1+\beta}{1-\beta}$$

$$9(1-\beta) = 1+\beta$$

$$10\beta = 8 \Rightarrow \boxed{\beta = \frac{v}{c} = 0.8}$$

(38-11) a) The rest energy of a proton is

$$m_p c^2 = 938 \text{ MeV}$$

The Lorentz factor for $v = 0.99c$ is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 7.09$$

The proton's total energy is ($1 \text{ GeV} = 10^3 \text{ MeV}$)

$$E = \gamma m_p c^2 = (7.09)(938 \text{ MeV}) = \boxed{6.65 \text{ GeV}}$$

Its kinetic energy is

$$K = E - m_p c^2 = 6.65 \text{ GeV} - 0.938 \text{ GeV}$$

$$K = \boxed{5.71 \text{ GeV}}$$

Its momentum comes from

$$E^2 = (pc)^2 + (m_p c^2)^2$$

or

$$\begin{aligned} pc &= \sqrt{E^2 - (m_p c^2)^2} \\ &= \sqrt{(6.65 \text{ GeV})^2 - (0.938 \text{ GeV})^2} \end{aligned}$$

$$pc = 6.58 \text{ GeV}$$

Thus $\boxed{p = 6.58 \frac{\text{GeV}}{c}}$

b) The electron's rest energy is $m_e c^2 = 0.511 \text{ MeV}$

$$\text{So } E = \gamma m_e c^2 = (7.09)(0.511 \text{ MeV}) = \boxed{3.62 \text{ MeV}}$$

$$K = E - m_e c^2 = 3.62 \text{ MeV} - 0.511 \text{ MeV}$$

$$K = \boxed{3.11 \text{ MeV}}$$

$$\begin{aligned} pc &= \sqrt{E^2 - (m_e c^2)^2} \\ &= \sqrt{(3.62 \text{ MeV})^2 - (0.511 \text{ MeV})^2} \\ &= 3.58 \text{ MeV} \end{aligned}$$

$$\text{Thus } \boxed{p = 3.58 \frac{\text{MeV}}{c}}$$

(38-12) The rest energy of the aspirin tablet is

$$\begin{aligned} E &= mc^2 = (320 \times 10^{-6} \text{ Kg})(3 \times 10^8 \frac{\text{m}}{\text{sec}})^2 \\ &= 2.88 \times 10^{13} \text{ J} \end{aligned}$$

This is equivalent to

$$\frac{2.88 \times 10^{13} \text{ J}}{1.3 \times 10^8 \text{ J/gal}} = 2.22 \times 10^5 \text{ gal of gasoline}$$

At $30 \frac{\text{mi}}{\text{gal}}$, a car could drive

$$30 \frac{\text{mi}}{\text{gal}} (2.22 \times 10^5 \text{ gal}) = \boxed{6.66 \times 10^6 \text{ miles!}}$$