HOMEWORK #10

(36-1) (a) $f = \frac{c}{\lambda} = \frac{3\times10^{8}\ \frac{m}{sec}}{589\times10^{-9}\,m} = \boxed{5.09\times10^{14}\ Hz}$

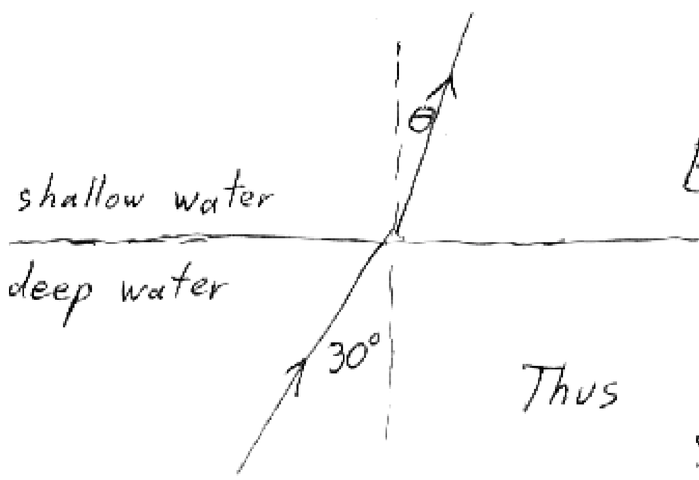
(b) $n_{air}\lambda_{air} = n_{glass}\lambda_{glass}$ (derived in class)

$$\lambda_{glass} = \frac{n_{air}}{n_{glass}}\lambda_{air}$$
$$= \frac{1}{1.52}\,589\,nm = \boxed{388\ nm}$$

(c) $V_{glass} = f\lambda_{glass}$

$$= \left(5.09\times10^{14}\ \frac{cycles}{sec}\right)\left(388\times10^{-9}\,m\right)$$
$$= \boxed{1.97\times10^{8}\ \frac{m}{sec}}$$

(36-2)



$n_{deep}\sin 30^\circ = n_{shallow}\sin\theta$

But $n = \frac{constant}{V}$, so

$$\frac{\sin 30^\circ}{V_{deep}} = \frac{\sin\theta}{V_{shallow}}$$

Thus

$$\sin\theta = \frac{V_{shallow}}{V_{deep}}\sin 30^\circ$$
$$\sin\theta = \frac{3\ m/sec}{4\ m/sec}\sin 30^\circ$$
$$\sin\theta = 0.375$$
$$\Rightarrow \boxed{\theta = 22.0^\circ}$$

(The constant in $n = \frac{constant}{V}$ is the wave speed in very deep water, but you don't have to know that!)

As $V_{shallow} \to 0$, $\theta \to 0^\circ$ so waves come in normal

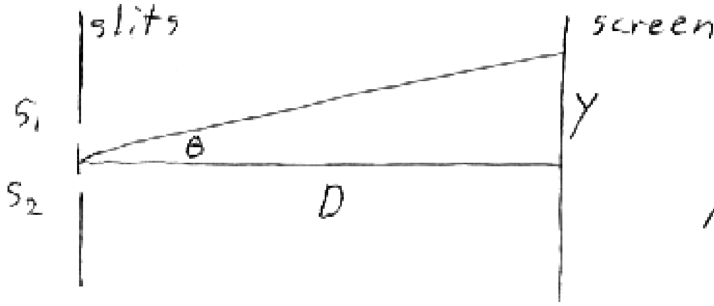
(36-3) (a) $d \sin\theta = m\lambda$ with $m = 3$

so $\sin\theta = \frac{m\lambda}{d} = \frac{3\,(550\times10^{-9}\,m)}{7.7\times10^{-6}\,m} = 0.2143$

$\Rightarrow \boxed{\theta = 0.216 \text{ radian}}$

(b) $\theta = (0.216\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right) = \boxed{12.4^\circ}$

(36-4)



Note that

$y = D\tan\theta \simeq D\sin\theta$

for small θ

The distance of the 5^{th} max from the central max is

$$y_5 \simeq D\sin\theta_5 = D\left(\frac{5\lambda}{d}\right)$$

The distance of the 7^{th} min from the central max is

$$y_7 \simeq D\sin\theta_7 = D\,\frac{(6+\frac{1}{2})\lambda}{d}$$

The distance between these is

$$y_7 - y_5 = 6.5\,\frac{D\lambda}{d} - 5\,\frac{D\lambda}{2}$$

$$= 1.5\,\frac{D\lambda}{d}$$

$$= 1.5\,\frac{(0.2m)(546\times10^{-9}m)}{0.1\times10^{-3}m}$$

$$= \boxed{1.64\times10^{-3}\,m}$$

(36–5) $2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$ for constructive interference

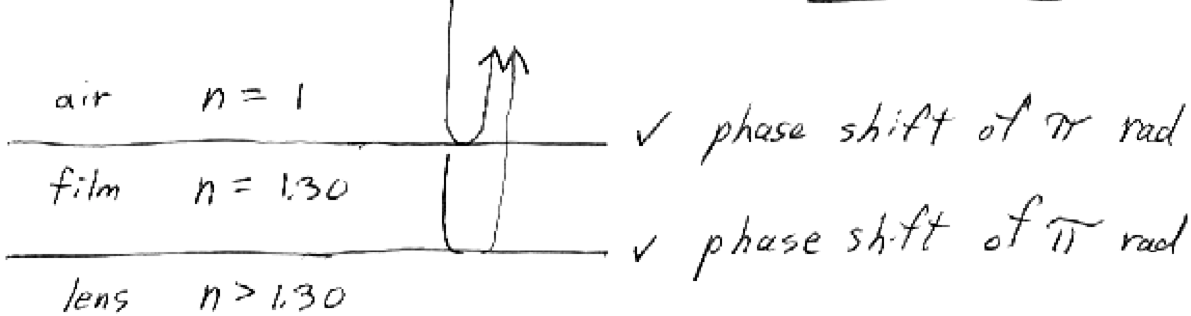
For $m = 0$,

$$L = \frac{1}{4}\frac{\lambda}{n} = \frac{1}{4}\,\frac{624\,nm}{1.33} = \boxed{117\,nm}$$

For $m = 1$,

$$L = \frac{3}{4}\frac{\lambda}{n} = \frac{3}{4}\,\frac{624\,nm}{1.33} = \boxed{352\,nm}$$

(36–6)



With two phase shifts of π radians, the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Using $m = 0$ for the minimum thickness,

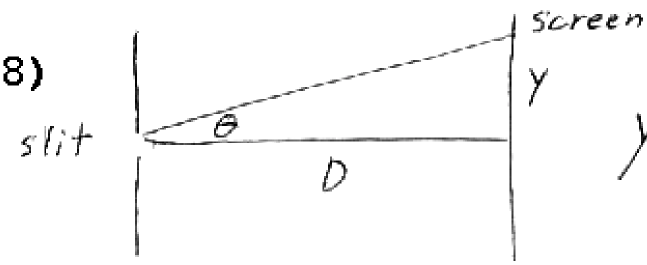
$$L = \frac{1}{4}\frac{\lambda}{n} = \frac{1}{4}\,\frac{680\,nm}{1.30} = \boxed{131\,nm}$$

(37–7) (a) $\tan\theta = \frac{1.5\times10^{-2}\,m}{2\,m} = 0.0075 \Rightarrow \boxed{\theta = 0.430^\circ}$

(b) $a\sin\theta = m\lambda$ with $m = 2$

So $a = \frac{m\lambda}{\sin\theta} = \frac{2(441\times10^{-9}\,m)}{\sin 0.43^\circ} = \boxed{1.18\times10^{-4}\,m}$

(37-8)



$y = D \tan\theta \simeq D \sin\theta$

for small θ

(a) The distance of the 5^{th} min from the central max is

$$y_5 \simeq D \sin\theta_5 = D\left(\frac{5\lambda}{a}\right) \qquad m=5$$

The distance of the 1^{st} min from the central max is

$$y_1 = D \sin\theta_1 = D\left(\frac{\lambda}{a}\right) \qquad m=1$$

So $\quad y_5 - y_1 = \frac{D\lambda}{a}(5-1) = 4\,\frac{D\lambda}{a}$

and therefore $\quad a = 4\frac{D\lambda}{(y_5 - y_1)}$

$$a = 4\,\frac{(0.4m)(550\times10^{-9}m)}{0.35\times10^{-3}m}$$

$$= \boxed{2.51\times10^{-3}m}$$

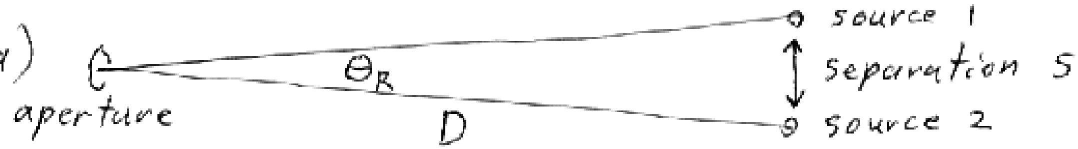
(b) $a \sin\theta = m\lambda$ with $m=1$

So $\quad \sin\theta = \frac{m\lambda}{a} = \frac{(1)(550\times10^{-9}m)}{2.51\times10^{-3}m} = 2.19\times10^{-4}$

$$\Rightarrow \boxed{\theta = 0.0126^\circ}$$

(37–9) $\theta_R = \dfrac{1.22\lambda}{d}$ (θ_R is in <u>radians</u>)

(a)



For small angles, $S = \theta_R D$, so

$$S = \left(\frac{1.22\lambda}{d_{eye}}\right) D$$

$$= \frac{(1.22)(550\times10^{-9}\,m)}{5\times10^{-3}\,m}(8\times10^{10}\,m)$$

$$= \boxed{1.07\times10^{7}\,m}$$

The diameter of Mars is only $6.8\times10^{6}\,m$, so your eye can not resolve any two objects on Mars!

(b) $$S = \left(\frac{1.22\lambda}{d_{Palomar}}\right) D$$

$$= \frac{(1.22)(550\times10^{-9}\,m)}{5.1\,m}(8\times10^{10}\,m)$$

$$= \boxed{1.05\times10^{4}\,m} \quad \text{or} \quad 10.5\,km$$

(37–10) $d\sin\theta = m\lambda$ with $m = 1$

Using $d = \dfrac{10^{-2}\,m}{10^{4}} = 10^{-6}\,m$, we find

$$\lambda = \frac{d\sin\theta}{m} = \frac{(10^{-6}\,m)\sin 30^\circ}{1} = 5\times10^{-7}\,m$$

or $\boxed{\lambda = 500\,nm}$

(37-11) The resolving power is $R = \frac{\lambda_{ave}}{\Delta\lambda} = Nm$

(a) $N = \frac{\lambda_{ave}}{m\Delta\lambda} = \frac{\frac{1}{2}(415.496 + 415.487)\,nm}{(2)(415.496 - 415.487)\,nm}$

$= \boxed{23,100}$

(b) $d \sin\theta = m\lambda$ with $m = 2$

So $d = \frac{4\times 10^{-2}\,m}{23,100} = 1.73\times 10^{-6}\,m$

and thus $\sin\theta = \frac{m\lambda}{d} = \frac{2(415.5\times 10^{-9}\,m)}{1.73\times 10^{-6}\,m} = 0.4803$

$\Rightarrow \boxed{\theta = 28.7^\circ}$

(37-12) $2d\sin\theta = m\lambda$ with $m = 2$

$d = \frac{m\lambda}{2\sin\theta} = \frac{(2)(0.12\times 10^{-9}\,m)}{2\sin 28^\circ}$

$\boxed{d = 2.55\times 10^{-10}\,m = 0.255\,nm}$