Homework #1

(22-1) The magnitude of the electric force is

a) $F = k \frac{|q_1||q_2|}{r^2}$

$= (8.99\times10^9 \frac{Nm^2}{C^2}) \frac{(1C)(1C)}{(1m)^2}$

$= \boxed{8.99\times10^9\ N}$ huge!

b) $F = (8.99\times10^9 \frac{Nm^2}{C^2}) \frac{(1C)(1C)}{(10^3 m)^2}$

$= \boxed{8990\ N}$

(22-2) $F = k \frac{|q_1||q_2|}{r^2}$ so $r = \sqrt{k \frac{|q_1||q_2|}{F}}$

So $r = \sqrt{(8.99\times10^9 \frac{Nm^2}{C^2}) \frac{(26\times10^{-6}C)(47\times10^{-6}C)}{5.7N}}$

$= \boxed{1.39\ m}$

(22-3) Use Newton's second law, $F_{net} = ma$, for particles 1 and 2:

$m_1 a_1 = k \frac{|q_1||q_2|}{r^2}$ and $m_2 a_2 = k \frac{|q_1||q_2|}{r^2}$

a) The forces have the same magnitude (as they must, from Newton's third law), so

$m_1 a_1 = m_2 a_2$

$\Rightarrow m_2 = m_1 \frac{a_1}{a_2} = (6.3\times10^{-7} kg) \frac{7\ m/sec^2}{9\ m/sec^2}$

$\boxed{m_2 = 4.9\times10^{-7} kg}$

b) The charges are equal, so $|q_1||q_2| = q^2$

Thus $m_1 a_1 = k \frac{q^2}{r^2}$

$$\Rightarrow \quad q = r\sqrt{\frac{m_1 a_1}{k}}$$

$$= (3.2\times10^{-3}\,m)\sqrt{\frac{(6.3\times10^{-7}\,Kg)(7\,m/sec^2)}{8.99\times10^{9}\,Nm^2/C^2}}$$

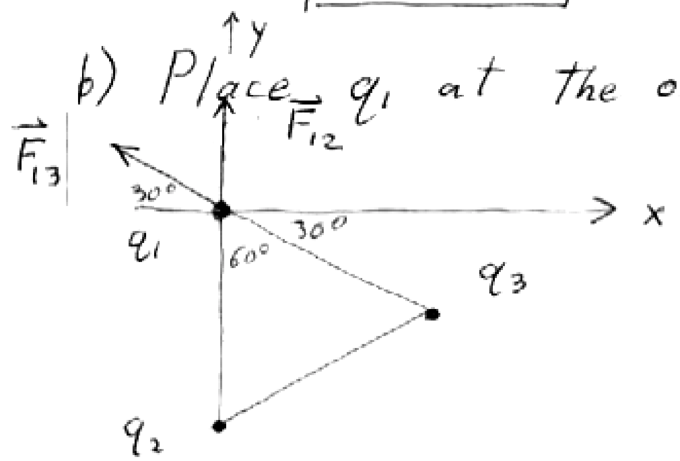
$$= \boxed{7.09\times10^{-11}\,C} \quad \text{magnitude}$$

(22-4) a) $F_1 = k\frac{|q_1||q_2|}{r^2}$

$$= \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(20\times10^{-6}\,C)(20\times10^{-6}\,C)}{(1.5\,m)^2}$$

$$= \boxed{1.60\,N}$$

b) Place q_1 at the origin. Draw vectors for the forces on q_1. Both have a magnitude 1.60 N. The components of F_{12} are



$F_{12,x} = 0 \quad F_{12,y} = 1.60N$

The components of $\vec{F}_{13}$ are

$F_{13,x} = -(1.60N)\cos 30° = -1.39\,N$

$F_{13,y} = (1.60N)\sin 30° = 0.80N$

So the total force on q_1 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

$$= (F_{12,x} + F_{13,x})\hat{i} + (F_{12,y} + F_{13,y})\hat{j}$$

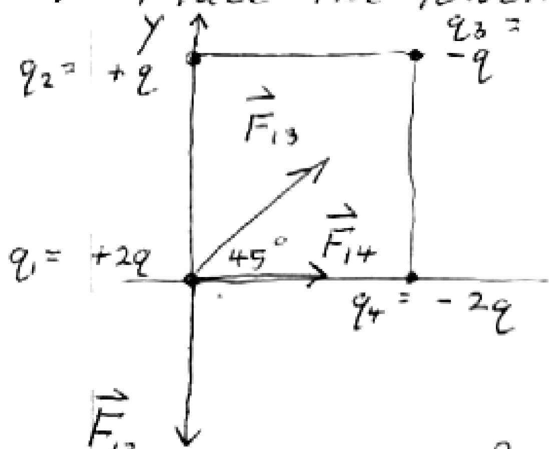
$$= (0 - 1.39\,N)\hat{i} + (1.60N + 0.8\,N)\hat{j}$$

$$= -1.39N\,\hat{i} + 2.40N\,\hat{j}$$

The magnitude of the force is

$$F_1 = \sqrt{(-1.39N)^2 + (2.40N)^2} = \boxed{2.77\,N}$$

(22-5) Place the lower left corner at the origin, and draw the forces on q_1. Note that the distances from q_1 are $r_{12} = r_{14} = a$ and $r_{13} = a\sqrt{2}$.



Find the components of each force:

$$F_{12} = k\,\frac{|q_1||q_2|}{r_{12}^2} = k\,\frac{(2q)(q)}{a^2}$$

$$\Rightarrow F_{12} = \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(2\times10^{-7}C)(10^{-7}C)}{(0.05m)^2}$$

$$= 7.192\times10^{-2}\,N$$

So $\boxed{\vec{F}_{12} = 0\hat{i} - 7.192\times10^{-2}N\,\hat{j}}$

Next, $$F_{13} = k\,\frac{|q_1||q_3|}{r_{13}^2} = k\,\frac{(2q)(q)}{(a\sqrt{2})^2}$$

$$\Rightarrow F_{13} = \left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(2\times10^{-7}C)(10^{-7}C)}{2\,(0.05m)^2}$$

$$F_{13} = 3.596 \times 10^{-2}\,N$$

So $$\vec{F}_{13} = (3.596 \times 10^{-2}\,N)\cos 45^\circ\,\hat{i} + (3.596 \times 10^{-2}\,N)\sin 45^\circ\,\hat{j}$$

$$\boxed{\vec{F}_{13} = 2.543 \times 10^{-2}\,N\,\hat{i} + 2.543 \times 10^{-2}\,N\,\hat{j}}$$

Next, $$F_{14} = k\,\frac{|q_1||q_4|}{r_{14}^2} = k\,\frac{(2q)(2q)}{a^2}$$

$$\Rightarrow F_{14} = \left(8.99 \times 10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(2\times 10^{-7}C)(2\times 10^{-7}C)}{(0.05\,m)^2}$$

$$= 1.4384 \times 10^{-1}\,N$$

So $$\boxed{\vec{F}_{14} = 1.4384 \times 10^{-1}\,N\,\hat{i} + 0\,\hat{j}}$$

Finally, find the total force on q_1:

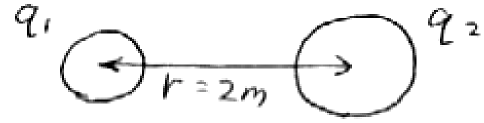
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$\vec{F}_1 = (0 + 2.543 \times 10^{-2}\,N + 1.4384 \times 10^{-1}\,N)\,\hat{i} + (-7.192 \times 10^{-2}\,N + 2.543 \times 10^{-2}\,N + 0)\,\hat{j}$$

$$\boxed{\vec{F}_1 = 1.693 \times 10^{-1}\,N\,\hat{i} - 4.65 \times 10^{-2}\,N\,\hat{j}}$$

horizontal component vertical component

(22.6)



$$q_1 + q_2 = 5 \times 10^{-5} C$$

The spheres repel, so q_1 and q_2 must both be positive since their sum is positive. Writing $q_2 = 5 \times 10^{-5} C - q_1$, we have

$$F = k \frac{|q_1||q_2|}{r^2} = k \frac{q_1 q_2}{r^2} = k \frac{q_1 (5 \times 10^{-5} C - q_1)}{r^2}$$

or $$\frac{Fr^2}{k} = (5 \times 10^{-5} C) q_1 - q_1^2$$

or $$q_1^2 - (5 \times 10^{-5} C) q_1 + \frac{Fr^2}{k} = 0$$

A quadratic! Using the quadratic formula

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{1}{2a}\left[-b \pm \sqrt{b^2 - 4ac}\right]$$

(memorize this!), we find

$$q_1 = \frac{1}{2}\left[5 \times 10^{-5} C \pm \sqrt{(5 \times 10^{-5} C)^2 - 4 \underbrace{\frac{(1N)(2m)^2}{8.99 \times 10^9 \frac{Nm^2}{C^2}}}_{\frac{Fr^2}{k}}}\right]$$

$$\boxed{q_1 = 3.84 \times 10^{-5} C \quad \underline{or} \quad 1.16 \times 10^{-5} C}$$

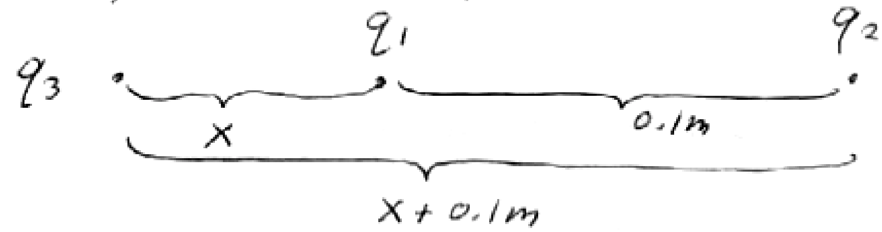
Then $q_2 = 5 \times 10^{-5} C - q_1$ is

$$\boxed{q_2 = 1.16 \times 10^{-5} C \quad \underline{or} \quad 3.84 \times 10^{-5} C}$$

(22.7) $q_1 = 1 \mu C$ • • $q_2 = -3 \mu C$

First, let's think. For the forces to cancel on q_3, all three charges must be in a straight line. Also, q_3 can't be between q_1 and q_2 because both forces on q_3 would be in the same direction and couldn't cancel.

So q_3 must be to the right or the left of both q_1 and q_2. Furthermore, it must be closer to the smaller magnitude charge.



We want $\quad F_{31} = F_{32} \quad$ equal magnitudes

$$k\frac{|q_3||q_1|}{x^2} = k\frac{|q_3||q_2|}{(x+0.1m)^2}$$

$$\frac{(x+0.1m)^2}{x^2} = \frac{|q_2|}{|q_1|} = \frac{3\mu C}{1\mu C} = 3$$

$$\frac{x+0.1m}{x} = \sqrt{3}$$

$$1 + \frac{0.1m}{x} = \sqrt{3}$$

$$\frac{0.1m}{x} = \sqrt{3} - 1$$

$$x = \frac{0.1\,m}{\sqrt{3}-1} = \boxed{0.137m}$$

from the $1\mu C$ charge.

(22.8) a) $F_{grav} = F_{electric}$ or $G\frac{M_E M_m}{r^2} = k\frac{q^2}{r^2}$

$$\Rightarrow q = \sqrt{\frac{G M_E M_m}{k}}$$

$$= \sqrt{\frac{(6.67\times10^{-11}\,Nm^2/Kg^2)(5.98\times10^{24}\,Kg)(7.36\times10^{22}\,Kg)}{8.99\times10^{9}\,Nm^2/C^2}}$$

$$= \boxed{5.71\times10^{13}\,C}$$

no way to collect so much charge!

b) Each hydrogen atom contributes one proton mass m_p and one fundamental charge e, where $e = 1.6\times10^{-19}\,C$

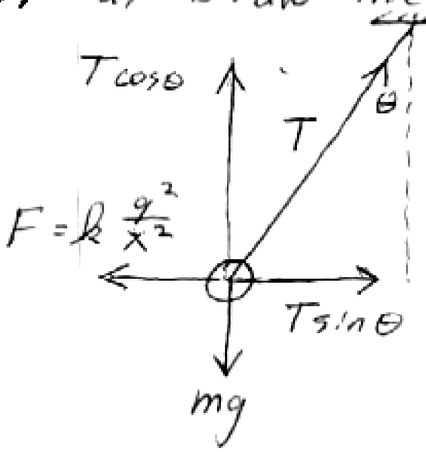
So the hydrogen mass required m_h is

$$m_h = \left(\frac{5.71\times10^{13}\,C}{1.6\times10^{-19}\,C}\right) m_p$$

$$= (3.57\times10^{32})(1.67\times10^{-27}\,kg)$$

$$= \boxed{5.96\times10^{5}\,kg}$$

(22.9) a) Draw the forces on one of the balls



For equilibrium,

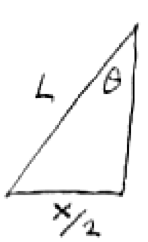
$$T\sin\theta = k\frac{q^2}{x^2}$$

$$T\cos\theta = mg$$

Divide these: $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{kq^2/x^2}{mg}$

or $\tan\theta = \dfrac{kq^2}{mgx^2}$

For small θ, $\tan\theta \simeq \sin\theta$ and $\sin\theta = \dfrac{x/2}{L}$



$$\Rightarrow \frac{x}{2L} = \frac{kq^2}{mgx^2}$$

or $x = \left(\dfrac{2kq^2L}{mg}\right)^{1/3}$

Writing $k = \dfrac{1}{4\pi\varepsilon_0}$, this is

$$\boxed{x = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}}$$

b) From $x^3 = \frac{2kq^2L}{mg}$,

$$q = \sqrt{\frac{mgx^3}{2kL}}$$

$$= \sqrt{\frac{(0.01\,kg)(9.8\,m/sec^2)(0.05\,m)^3}{2(8.99\times10^9\,Nm^2/C^2)(1.2\,m)}}$$

$$= \boxed{\pm\, 2.38\times10^{-8}\,C}$$

(22.10) $F = k\frac{|q_1||q_2|}{r^2} = (8.99\times10^9\,\frac{Nm^2}{C^2})\frac{(1.6\times10^{-19}C)^2}{(2.82\times10^{-10}m)^2}$

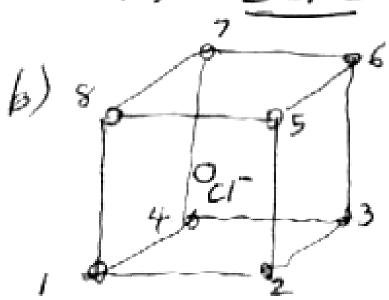
$$= \boxed{2.89\times10^{-9}\,N}$$

(22.11) a) $F = k\frac{|q_1||q_2|}{r^2} = (8.99\times10^9\,\frac{Nm^2}{C^2})\frac{(10^{-16}C)^2}{(0.01m)^2}$

$$= \boxed{8.99\times10^{-19}\,N}$$

b) $\#\,\text{electrons} = \frac{q}{-e} = \frac{-10^{-16}C}{-1.6\times10^{-19}C} = \boxed{625\text{ electrons}}$

(22.12) a) By symmetry, the total force on the Cl^- ion is zero.

b) 

The force on the Cl^- ion with all eight Cs^+ ions is

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 + \vec{F}_7 + \vec{F}_8 = 0$$

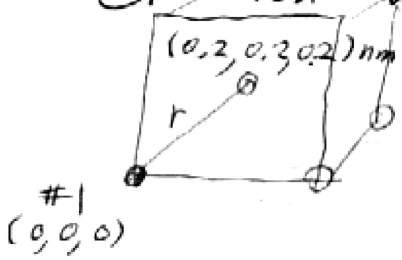
Let's remove Cs^+ ion #1. Then we must calculate the vector sum

$\vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 + \vec{F}_7 + \vec{F}_8$ yuck!

But this is <u>easy</u>, since

$$\vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 + \vec{F}_7 + \vec{F}_8 = -\vec{F}_1$$

So the magnitude of the force on the Cl^- ion with Cs^+ ion #1 missing is



$$F_1 = k\frac{e^2}{r^2}$$

where $r = \sqrt{(0.2\text{ nm})^2 + (0.2\text{ nm})^2 + (0.2\text{ nm})^2}$

$$= \sqrt{3(0.2\text{ nm})^2} = 0.2\text{ nm}\sqrt{3}$$

So $$F_1 = \left(8.99\times10^9\,\frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(1.6\times10^{-19}\text{C})^2}{[(0.2\times10^{-9}\text{m})\sqrt{3}]^2}$$

$$= \boxed{1.92\times10^{-9}\text{ N}}$$