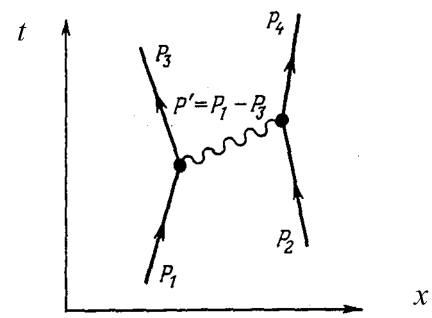
Feynman Diagrams

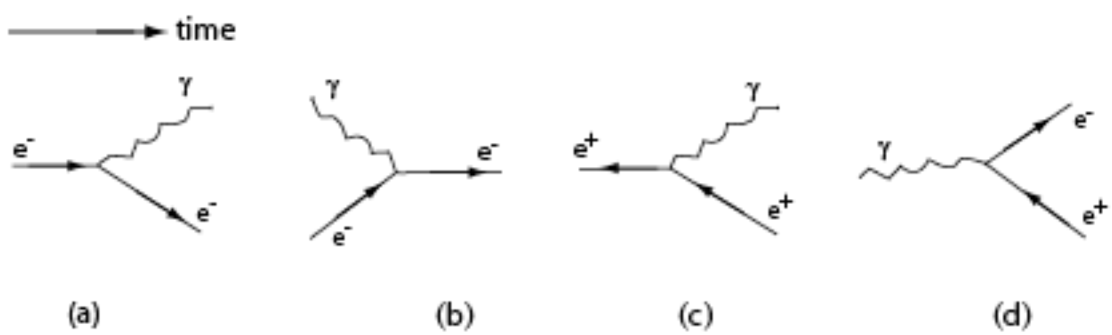
Richard Feynman developed this method for simplifying the quantitative calculations in electromagnetic phenomena. Later, this method was also applied for a qualitative description of weak and strong interactions.

It was shown by Feynman in 1949 that without any loss of accuracy, the complex and cumbersome methods of calculations in quantum electrodynamics can be replaced by a graphic method in which any electromagnetic process is represented by a diagram that is then analyzed mathematically in a comparatively simple way by adopting standard procedure.



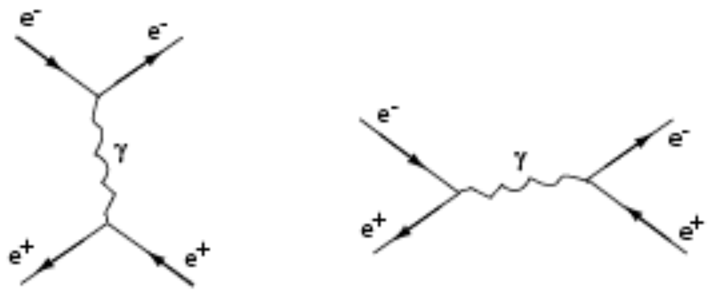
According to Feynman, the electromagnetic interaction between two charges e_1 and e_2 (for example, the scattering of an electron by an electron) can be represented on the (x, t) coordinate plane in the form of the diagram shown. Here, the *outer* broken lines represent the world lines of interacting charged particles before and after the interaction. According to the law of conservation of electric charges, the outer lines do not suffer a discontinuity anywhere. They start at $-\infty$ and terminate at $+\infty$. The slope of a line relative to the *t*-axis can characterize the momentum of the electron. The *inner* wavy line represents a virtual photon. The interaction itself is represented by the point of intersection of an outer line with the inner one (*vertex* of the diagram). Usually the Feynman diagram shows only the direction of the *t*-axis (in our case, this direction is upwards) and the direction of motion of the particle relative to this. The rest of the construction is arbitrary, for example, the slope of the line relative to the *t*-axis is arbitrary.

The vertices are the points of interactions, as shown in the following diagrams.



(a) photon emission by an electron
(b) photon absorption by an electron
(c) photon emission by a positron
(d) pair production of an electron and a positron

The vertex rules are that energy and momentum are conserved, and the electric charge is conserved as well. The amplitude of every diagram contains several factors. The vertex factor is related to the strength of the interaction, the electric charge "e" in the above examples. Usually this factor is written in terms of the electromagnetic fine structure $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$. Thus each vertex is represented by the factor $\sqrt{\alpha}$. As an example, the following figures show the leading-order Feynman diagram for electron-positron elastic scattering (notice that time is to the right).



Each vertex contributes a factor of $\sqrt{\alpha}$, and the amplitude of this process is proportional to $\sqrt{\alpha} \times \sqrt{\alpha} = \alpha$. Therefore, it is proportional to "e^2" as it should be.

As was mentioned above, this method assumes that the electromagnetic interaction involves the exchange of one photon. Sometimes, the accuracy of such a single-photon approximation is found to be sufficient. However, the actual nature of electromagnetic interaction is much more complicated than this approximate description. Charged particles may exchange not just one photon, but even 2 or more photons. Hence, in many cases a single-photon approximation may not have sufficient accuracy as compared to the potentialities of the modern experiment. In such cases, besides the lower-order diagram, higher-order diagrams must also be taken into account. The consideration of additional contribution from higher-order diagrams is called *radiative* correction. Figure 2

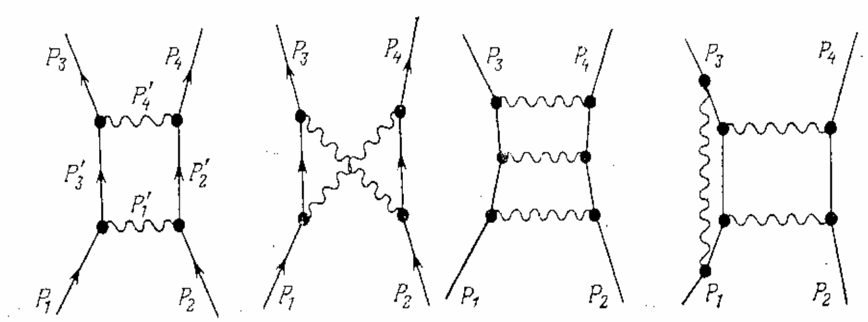
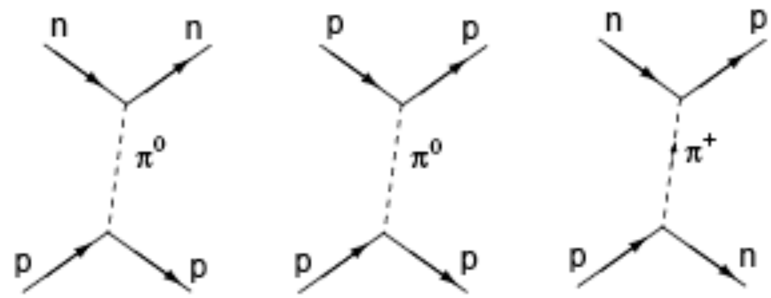


Fig. 2

Fig. 3

shows two examples of fourth-order diagrams for (*e*-*e*)-scattering.

The method of Feynman diagrams can be also used for describing strong nuclear interaction. The diagrams are constructed as before, although the outer broken lines now represent the interacting nucleons, while the inner dashed line represents the virtual π-meson. As before, the outer lines originate at $-\infty$ and terminate at $+\infty$, without suffering any discontinuity (the law of conservation of electric charge). As before, a vertex describes the process of interaction, although the intensity of interaction is characterized by the meson "charge" g_N of the nucleon, and not by the electric charge e.



It is very important to note that the dimensionless quantity $f = g_N^2 / \hbar c$, obtained from g_N in analogy with the fine structure constant $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137}$, is found to be of the order of unity, i.e., $f = g^2{}_N / \hbar c \approx 1$. Its value can be estimated by comparison with the experiment.

This means that the contribution to the interaction amplitude from the higher-order diagrams (proportional to f^2, f^3, etc.) is comparable with the contribution from the lower-order diagrams. All the diagrams become equally important and all the terms in the series have the same order. The series becomes divergent and it is not possible to make any calculations. This is the main difficulty associated with the meson theory. The origin of this difficulty is linked with the high intensity of nuclear interaction.

In the case of electromagnetic interaction, the small value $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$ indicates a low density of the virtual photon cloud surrounding the electric charge (comparatively low emission frequency), i.e. a relatively weak interaction with other charges. In this case, the interaction involving the exchange of one photon has a much higher probability than the two-photon (not to speak of three- or four-photon) exchange interaction. Conversely, in the case of strong interaction, $f = g^2{}_N / \hbar c \approx 1$ indicates a very high density of the meson cloud surrounding the nucleon (virtual mesons are emitted frequently), and the multimeson exchange has nearly the same probability as the single-meson exchange.

Nonetheless, the case of meson theories cannot be considered as utterly hopeless. It can be shown that by imposing certain restrictions on the phenomena under consideration we can construct a semiquantitative meson theory capable of explaining many features of strong interaction.

These restrictions are based on a comparison with the experimentally established properties of nuclear forces. For example, the fact that nucleons in a nucleus (i.e. at low interaction energies) do not come closer than 2×10^{-13} cm indicates that the multimeson exchange (which has a smaller range than the single-meson exchange) is not very significant in this case.

In this theory, the following value is obtained for the dimensionless pion-nucleon (π-*N*) interaction constant:

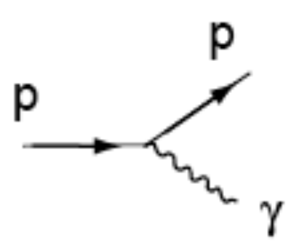
$$f^2 = 0.08.$$

The smallness of this quantity relative to unity allows us to consider some phenomena in the single-pion-exchange approximation. For example, this method was used for considering (*N*-*N*)- and (π-*N*)-scattering, and even for estimating the (π-π)-scattering cross section which cannot be determined experimentally on account of the fact that π-meson targets and colliding π-meson beams are not available.

Questions

1. Draw all possible leading order diagrams for electron-electron elastic scattering. Make sure to label each line with the momentum of the respective particle.

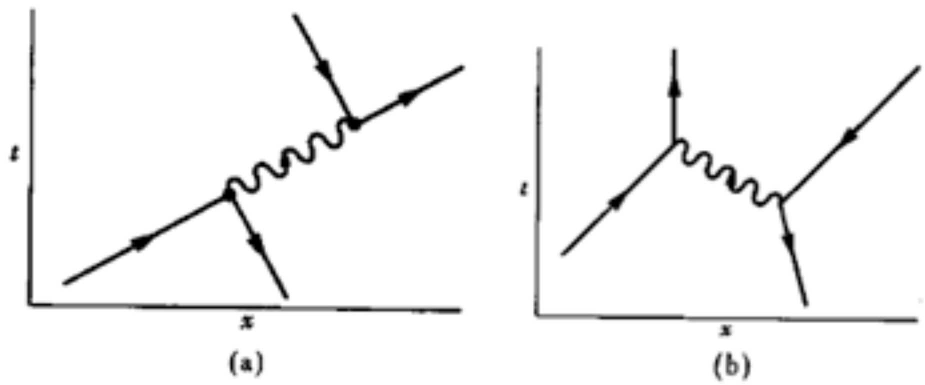
2. The following figure shows a free proton moving with an energy $E >> mc^2$, emitting a photon and moving in a different direction. Apply the laws of conservation of energy and momentum to prove that this process is impossible if all three particles are free.



3. Draw the two leading-order diagrams for the production of an electron-positron pair by a real photon in the electric field of a nucleus.

4. Use the law of conservation of angular momentum applied to a vertex point, to prove the exchange particle (like a photon, or pions) must have integer spins. That is the exchanged particles, or the carriers of the “force”, or field quanta, are bosons.

5. For the diagrams shown, describe the sequence of events.



6. A 50-MeV pion decays into two photons:

$$\pi^0 \rightarrow \gamma + \gamma$$

If the photons move parallel and antiparallel to the direction of motion of the pion, what will be the energy of each photon?