Introduction to Nuclear Forces

One of the main problems of nuclear physics is to find out the nature of nuclear forces. Nuclear forces differ from all other known types of forces. They cannot be of electrical origin since they act between charged particles as well as neutral particles (say, between a neutron and a proton in a deuteron). These forces cannot be magnetic either because the interaction between the magnetic moments of the nucleons is extremely weak. Thus nuclear interactions cause by forces, which differ from all known types of forces and are called nuclear forces. Let us recall the main properties of nuclear interactions.

Most of the information about forces among nucleons is obtained from the study of a simple two-nucleon system like deuteron. The ground state of the deuteron is characterized by the following measured quantities.

- Binding energy: $\Delta E = 2.22 \, MeV$

- Nuclear spin: $J = 1$

- Even parity

- Magnetic dipole moment: $\mu = +0.857 \, \mu_n$

- Electric quadrupole moment: $Q = +2.7 \times 10^{-31} \, m^2$

- Charge distribution half-value radius: $a = 2.1 \, fm$

The fact that the deuteron has an electric dipole moment Q indicates that its probability density function is not spherically symmetrical. This immediately tells us that the nuclear potential, which specifies the force acting between the two nucleons is, itself, not spherically symmetric. The point is that all spherically symmetric potentials have $l = 0$ eigenfunctions for their ground states, indicating a zero quadrupole moment for their charge distributions.

Calculations show that the measured electric quadrupole moment is obtained if the ground state if the deuteron is a mixture in which 96% is a $l = 0$ state and 4% is a $l = 2$ state. In spectroscopic notation, the dominant state is $^3S_1$ and the less probable state is $^3D_1$. Calculations also show that this mixture of states lead to the measured magnetic dipole moment $\mu = +0.857 \, \mu_n$. The value differs by 3% from what would be obtained if the deuteron were in a pure $^3S_1$ state, i.e. $\mu = \mu_P + \mu_N = +2.7896 \, \mu_n - 1.9103 \mu_n = +0.8793 \, \mu_n$. We conclude from all these considerations that the nuclear potential is not precisely spherically symmetric, since it does not lead to a pure S ground state for the deuteron.

Summary of Properties of Nuclear Forces

1. Nuclear forces are forces of attraction, as can be seen from the existence of stable nuclei consisting of protons and neutrons.

2. Nuclear forces are short range. Rutherford's experiment on the scattering of alpha particles by nuclei showed that up to distances of about 10^{-12} cm, the experimental results can be explained by assuming that the interactions between alpha particles and nuclei is purely of Coulomb type. This means that the nuclear forces are short range and their range can be estimated as the average distance between nucleons that are bound within the nucleus by nuclear forces.

$$a \approx (V / A)^{1/3}$$
$$but\ R = R_0 A^{1/3}, with\ R_0 = 1.1\ fm$$
$$a \approx (\frac{4}{3}\pi R^3 / A)^{1/3} \cong 2 \times 10^{-13} cm = 2\ fm$$

3. The constant value of the average binding energy per nucleon for most nuclei indicates that the nuclear forces have the property of saturation.

4. Nuclear forces are spin dependent. We know that there are no bound deuteron with nucleon spins essentially antiparallel, i.e. in a state of $^0 S_1$. This indicates that the nuclear potential is spin dependent, being appreciably weaker when two nucleons interact with their spins antiparallel (in a singlet state).

5. The similarity in the level structure of some light nuclei leads to the hypothesis of charge independence (isotopic invariance) of nuclear forces. The concept of isotopic invariance will be discussed later.

The Concept of the Meson Theory of Nuclear Forces

The meson theory of nuclear forces is constructed in analogy with quantum electrodynamics. It is well known that in quantum electrodynamics the electromagnetic field is considered jointly with the particles (photons) associated with it. The field as if consists of photons which are the quanta of this field. The field energy is equal to the sum of the energies of the quanta. Photons are created (annihilated) during emission (absorption) of electromagnetic radiation (say, light). The electric charge is the source of photons. The interaction between two charges is responsible for the emission of a photon by one charge and its absorption by the other. Such an approach makes it possible to consider new phenomena associated with the interaction of radiating systems with the intrinsic radiation field. It explains, for example, the anomalous magnetic moment of the electron and the muon, Lamb's shift of levels in the fine structure of the hydrogen atom, and many other fine effects.

The basic idea of quantum electrodynamics, viz. the quantum nature of interactions, can be also extended to other types of interaction, including nuclear interaction. This idea was first put forth by L. Tamm in 1934. Tamm's idea provided a clear graphic interpretation for such properties of the nuclear interaction as its exchange nature which can be explained by assuming that the proton and the neutron exchange charge during their interaction and that this leads to saturation. It seemed quite natural to assume that the exchange mechanism involves the transfer (at the instant of nuclear interaction) of some light particles from one nucleon to the other. These particles can be, for example, electrons or neutrinos. However, it was shown later by Tamm that

these particles cannot be quanta of the nuclear field; since they cannot simultaneously explain the small range of nuclear forces and the high binding energy. No other light particles were known at that time.

Tamm's idea was later developed by the Japanese physicist Yukawa who assumed (in 1935) that the role of nuclear quanta is played by unstable charged or neutral particles, the mesons, which had not been experimentally discovered at that time, but which were supposed to have a mass $m \approx 200\,m_e$. Yukawa's arguments can be graphically presented as follows. According to quantum mechanics, there exists the uncertainty relation

$$\Delta E \Delta t \approx \hbar$$

Putting $\Delta E = mc^2$, we can assume that the energy $\Delta E = \hbar / \Delta t$ may be responsible for the creation of a virtual meson with mass $m = \Delta E / c^2 = \hbar / c^2 \Delta t$ for a short time Δt in the immediate vicinity of the nucleon.

Unlike ordinary particles that can move freely in space and in time, virtual particles exist only for a short time Δt during which they must be separated from the nucleon by a distance a not exceeding $a = c \Delta t$. After the passage of time Δt, the virtual particle is "captured" once again by a nucleon. Thus, it can be assumed that a nucleon is surrounded by a cloud of virtual mesons that are continuously being created and annihilated. The radius of this meson cloud is given by

$$a = c\hbar / \Delta E = \hbar / mc.$$

A virtual meson can be absorbed not only by its "own" nucleon, but also by some other nucleon if it happens to be in the meson cloud of the latter. It is this transfer of a virtual meson from one nucleon to another that is responsible for the nuclear interaction.

Quantitative estimates for the nuclear interaction time τ_{nuc} and the virtual meson mass m can be easily obtained by equating a to the range of nuclear forces. Assuming this value to be 2×10^{-13} cm (latest estimates put this value at 1.4×10^{-13} cm), Yukawa obtained

$$\tau_{nuc} = \Delta t = a/c = 0.7 \times 10^{-23} \text{ s},$$

$$\Delta E \approx \frac{\hbar}{\Delta t} \cong 100 \ MeV, \ m \cong 200 m_e$$

This is how the nuclear quantum or the Yukawa meson was predicted.

If Yukawa mesons do exist in actual practice, they can be detected only if they are created in a free state and not virtually, i.e., if they are separated from the place of their origin by a distance exceeding the range of nuclear forces. Such a process is possible only when the law of energy conservation is obeyed. Hence, the creation of mesons requires a large kinetic energy of the colliding nucleons, a part of which may be transformed into the rest energy of the created mesons.

The discovery of nuclear quanta is associated with an interesting and instructive course of events. It was first decided that the μ-mesons (now called muons) with a mass $m = 207\ m_e$, which were detected in 1938 in cosmic rays, are the nuclear quanta. However, it was soon found that muons do not participate in a strong nuclear interaction. Later, in 1947-50, pions or $\pi - mesons$ were detected first in cosmic rays and then in accelerators. Pions (π^+, π^-, and π°-mesons) are strongly interacting particles with a mass of approximately $m = 270m_e$.

It is the pions that play the role of nuclear quanta (probably together with some other strongly interacting particles). It can be easily seen that for $m = 270m_e$ (corresponding to $\Delta E \approx 140 MeV$)

$$\Delta t \approx \frac{\hbar}{\Delta E} = 0.5 \times 10^{-23}\ s, \quad a = \frac{\hbar}{mc} = 1.4 \times 10^{-13}\ cm$$

The discovery of π-mesons stimulated the development of specific versions of meson theories taking into account the properties of nucleons and π-mesons. We cannot go into details of these theories, and shall confine ourselves to just the rough semiqualitative concepts of the meson theory obtained in analogy with quantum electrodynamics.

Applying Quantum Mechanics to Mesons

It was mentioned above that according to quantum electrodynamics, the mechanism of electromagnetic interaction involves the transfer of a photon from one charge to another. The equation of motion for a freely moving photon can be written in the form

$$E^2 = p^2 c^2.$$

In order to obtain the equation for the potential field of a unit charge, we must make the substitution

$$E \rightarrow -\frac{\hbar}{i}\frac{\partial}{\partial t}; \qquad p \rightarrow \frac{\hbar}{i}\nabla.$$

The equation for the potential in empty space will then assume the form

$$\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = 0.$$

For the time-independent case, $\frac{\partial}{\partial t}\psi = 0$ and the solution of the above equation is the function

$$\psi = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

This of course can be verifies by substituting the solution in the differential equation and taking note of

$$\nabla^2 \psi = \frac{1}{r^2}\frac{d}{dr}(r^2 \frac{d}{dr}\psi)$$

The solution $\psi = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$ is the expression for the interaction potential energy of a unit charge

(-e) in the potential V given by $V = \frac{e}{4\pi\varepsilon_0 r}$.

It follows from the above analysis (which obviously coincide with the corresponding expressions in electrostatics) that the electromagnetic interaction has an infinitely long range.

We have mentioned earlier that according to meson theories, the transfer of interaction takes place through a π-meson that is a particle with a nonzero mass ($m \neq 0$). The equation for a freely moving particle with $m \neq 0$ is written in the form

$$E^2 = p^2c^2 + m^2c^4.$$

After the substitution for the energy and momentum operators, the equation for the meson potential field of a nucleon in empty space assumes the form

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\varphi = 0.$$

For the time-independent case ($\partial\varphi/\partial t = 0$), the solution of this equation has the form

$$\varphi = -g_N^{\,2}\frac{e^{-\frac{r}{\lambda}}}{r}$$

In the above equation, $\lambda = \frac{\hbar}{mc}$ and $\frac{g_N^{\,2}}{\hbar c}$ is the strength of the Yukawa potential. It plays a role

similar to the dimensionless quantity $\frac{e^2}{4\pi\varepsilon_0\hbar c}$ for electromagnetic interactions. The magnitude of

the "charge" g_N can be determined from a comparison with the experiment. The wave function φ is related to the meson field surrounding a nucleon. This rapidly decreasing function

$V = -g_N\dfrac{e^{-\frac{r}{\lambda}}}{r}$ is called the Yukawa potential.

The Compton wavelength $\lambda = \hbar/mc$ of the meson can serve as a measure of the rate at which the function φ decreases (i.e. a measure of the "radius" of the meson cloud). For $m_\pi = 270m_e$ we get

$$\lambda_\pi^{Com} = 1.4\times10^{-13} \text{ cm.}$$

Apparently, the quantity $\lambdabar_\pi^{\text{Com}}$ coincides with the range a of the nuclear forces introduced above:

$$a = c\Delta t = \frac{c\hbar}{\Delta E} = \frac{\hbar}{m_\pi c} = \lambdabar_\pi^{\text{Com}}.$$

Experimental Verifications

Experimental evidence for the exchange of pions between two interacting nucleons is found in neutron-proton scattering. The first high energy experiment was performed with incident neutrons of energy 90 MeV. The measurements show that the differential cross section $\dfrac{d\sigma}{d\Omega}$ is approximately symmetric about a scattering angle of 90°.
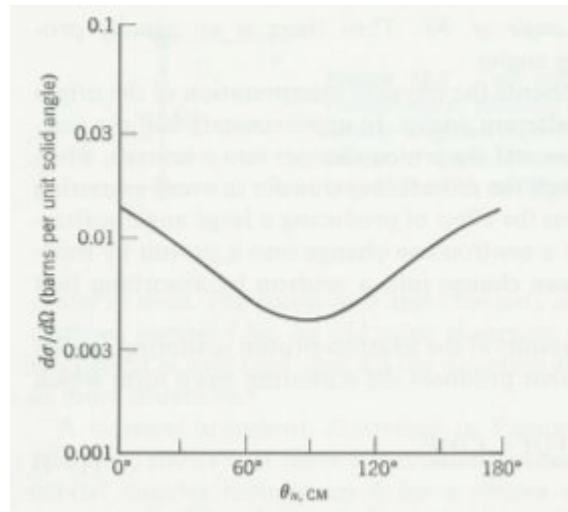
Thus, there is an equally pronounced preference for large scattering angles. The physical interpretation of the observed preference of large angles is that in approximately half the scattering, the neutron changes into a proton and the proton changes into a neutron, when the two nucleons are very close. One way this can happen is indicated by the set of reactions:

$$n \rightarrow p + \pi^- \quad then \quad \pi^- + p \rightarrow n$$
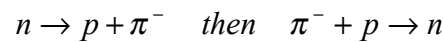
That is, the neutron emits a negatively charged π^- meson into its field, becoming a proton. Then the π^- meson joins the field of the proton, and it is absorbed by the proton, which becomes a neutron. The same scattering process can also happen through the set of reactions

$$p \rightarrow n + \pi^+ \quad then \quad \pi^+ + n \rightarrow p$$

In this case, the proton emits a positively charged π^+ meson, which is subsequently absorbed by the neutron. Thus, in about half the neutron-proton scatterings, a meson transfers charge as well as momentum between the two interacting nucleons.

In about half of the scatterings, the neutrons and protons do not exchange identities when they interact but they still must exchange a meson that carries the transferred momentum. The two sets of reactions that occur are

$$n \rightarrow n + \pi^0 \quad then \quad \pi^0 + p \rightarrow p$$
$$p \rightarrow p + \pi^0 \quad then \quad \pi^0 + n \rightarrow n$$
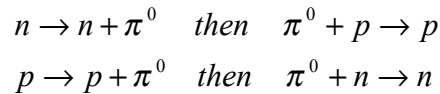
The neutral π^0- meson transfers momentum but no charge between the interacting nucleons.

This picture implies that an isolated proton should be surrounded by a meson field which will sometimes contain a π^0- meson and sometimes contain a π^+- meson. Of course, the nucleon must absorb the meson it has emitted within a very short time, but then it can emit another one. Similarly, the meson field surrounding an isolated neutron should sometimes contain a π^0- meson and sometimes a π^-- meson. But the proton files cannot contain a π^-- meson and the neutron field cannot contain a π^+- meson.

Experimental verification of these predictions is provided by electron scattering measurements of the charge distribution of the proton and of the neutron. The following figure shows the radial dependence of the charge densities of the two species of nucleons. The charge density of the proton is everywhere positive, and extends out to a distance r of about 2 fm. At larger r, this charge is carried by a π^+- meson. For the neutron, the charge density is not everywhere zero. At smaller r, it is positive and at larger r it is negative. The volume integral of the charge density is however zero since the neutron is neutral and has no net charge.
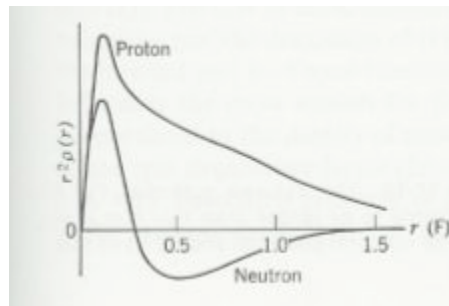
At values of r approaching 2 fm, the nucleon charge densities are proportional to some measure of the intensity of their meson field. Both proton and neutron charge densities are decreasing fairly gradually as r increases. The nucleon force, that acts between two nucleons when their meson fields overlap, also therefore decreases gradually as their separation increases. Thus the onset of the attractive part of the nuclear potential, describing the nucleon force acting when the two nucleons are beginning to get close enough to interact, is fairly gradual and it is not as depicted in the following figure. Nevertheless, this is a good approximation for description of many features of nuclear potential.
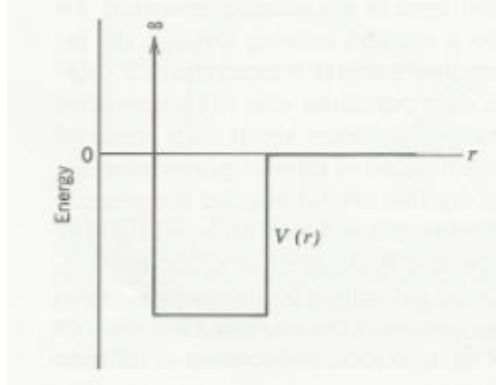
Meson theory also provides an explanation of how the neutron can have an intrinsic magnetic dipole moment, even though its net charge is zero. The neutron sometimes become a proton plus a π^- - meson. The proton has an intrinsic magnetic moment, and the π^- - meson can produce a current that makes an additional contribution to the magnetic dipole moment.

Questions

1. Why is $^3P_1$ not a component of the ground state of the deuteron? What about $^1S_0$?

2. Explain why a stable system of two neutrons has not been observed?

3. What particle would remain if a proton emitted a π^- - meson? If a neutron emitted a π^+ - meson? Why is it that a proton field cannot contain only a π^- - meson and a neutron field contains only a π^+ meson?

4. Estimate the maximum time that a π meson can exist in the field of an isolated nucleon before it is absorbed by that nucleon. Estimate how many π mesons there can be at any instant in the field of a nucleon at a distance of 2 fm and the distance of 0.5 fm.

5. The π^0 lifetime has been determined by studying the decay $K^+ \rightarrow \pi^0 + \pi^+$ from rest. The average distance traveled by the π^0 in the block of photographic emulsion before it decays in the observable mode $\pi^0 \rightarrow e^+ + e^- + \gamma$ is measured, and from the calculated velocity of flight of the π^0, its lifetime is obtained. Given that the lifetime is 0.8×10^{-16} s, predict the average distance traveled by a π^0 before it decays.