1.2

a) $\vec{A}\cdot(\vec{B}+\vec{C}) = (2\hat{\imath}+\hat{\jmath})\cdot(\hat{\imath}+4\hat{\jmath}+\hat{k})$

$= 2+4 = 6$

$(\vec{A}+\vec{B})\cdot\vec{C} = (3\hat{\imath}+\hat{\jmath}+\hat{k})\cdot 4\hat{\jmath} = \cdots = 4$

b) $\vec{A}\cdot(\vec{B}\times\vec{C}) = (2\hat{\imath}+\hat{\jmath})\cdot\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 0 & 1\\ 0 & 4 & 0\end{vmatrix}$

$= (2\hat{\imath}+\hat{\jmath})\cdot(-4\hat{\imath}+4\hat{k})$

$= -8+0 = -8$

Similarly $(\vec{A}\times\vec{B})\cdot\vec{C} = \cdots = -8$

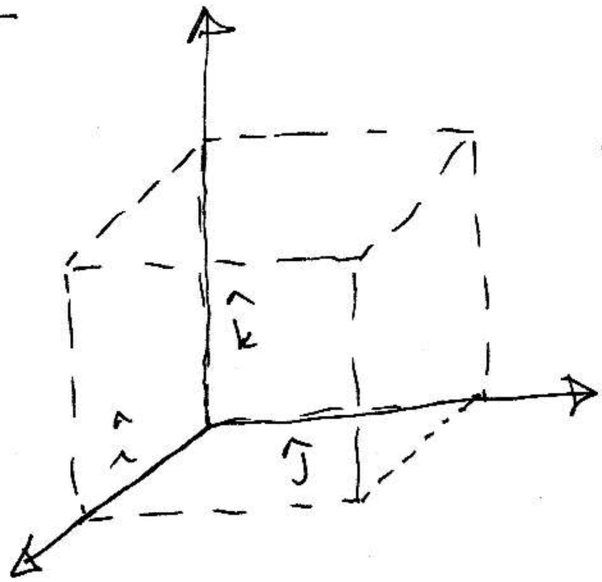
c) $(\vec{A}\times\vec{B})\times\vec{C} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & 0\\ 0 & 4 & 0\end{vmatrix}\times 4\hat{\jmath} = 4\hat{\imath}+4\hat{k}$

and

$\vec{A}\times(\vec{B}\times\vec{C}) = (2\hat{\imath}+\hat{\jmath})\times\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 0 & 1\\ 0 & 4 & 0\end{vmatrix}$

$= 4\hat{\imath}-8\hat{\jmath}+4\hat{k}$

1.4



a) Let's call $\vec{A}$ = Body Diagonal

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

Thus:

$$\vec{A}\cdot\vec{A} = A^2 = \hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{k} = 3$$

$$\boxed{A = \sqrt{3}}$$

b) $\vec{B} = \hat{i} + \hat{j}$ = face Diagonal

$$\vec{B}\cdot\vec{B} = B^2 = \hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} = 2 \Rightarrow \boxed{B = \sqrt{2}}$$

c) $$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$$

d) $$\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\cos\theta = \frac{1+1}{\sqrt{3}\,\sqrt{2}} = \sqrt{2/3} \rightarrow$$

$$\boxed{\theta = 35.3^\circ}$$

1.9

We manipulate the left hand side to get the right hand side.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= \vec{A} \times \left[\hat{\imath}(B_y C_z - B_z C_y) + \hat{\jmath}(B_z C_x - B_x C_z) + \hat{k}(B_x C_y - B_y C_x) \right]$$

$$= (A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}) \times [\ \cdots\ ]$$

Using $\hat{\imath} \times \hat{\imath} = 0 = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k}$

$\hat{\imath} \times \hat{\jmath} = \hat{k}$, etc. $\Rightarrow$

$$= \hat{k}\left[A_x(B_z C_x - B_x C_z)\right] - \hat{\jmath}\left[A_x(B_x C_y - B_y C_x)\right]$$
$$- \hat{k}\left[A_y(B_y C_z - B_z C_y)\right] + \hat{\imath}\left[A_y(B_x C_y - B_y C_x)\right]$$
$$+ \hat{\jmath}\left[A_z(B_y C_z - B_z C_y)\right] - \hat{\imath}\left[A_z(B_z C_x - B_x C_z)\right]$$

We regroup the above terms according to the terms on R.H.S.

$=$

1.9 Continues

$$= \hat{i} B_x \left[A_y C_y + A_z C_z\right] + \hat{j} B_y \left[A_x C_x + A_z C_z\right] + \hat{k} B_z \left[A_x C_x + A_y C_y\right]$$

$$- \hat{i} C_x \left[A_y B_y + A_z B_z\right] - \hat{j} C_y \left[A_x B_x + A_z B_z\right] - \hat{k} C_z \left[A_x B_x + A_y B_y\right]$$

Notice that there is a "missing" term in each [...]. Thus, we "add" and "subtract" the needed term. For example, take the first [...]:

$$\hat{i} B_x \left[A_y C_y + A_z C_z \overbrace{+ A_x C_x}^{\text{add}} \overbrace{- A_x C_x}^{\text{subtract}}\right]$$

$$= \hat{i} B_x \left[\underbrace{A_y C_y + A_z C_z + A_x C_x}_{\vec{A} \cdot \vec{C}}\right] - \hat{i} C_x (A_x B_x)$$

This is the term missing from $\hat{i} C_x [\ldots]$. Thus

$$= \hat{i} B_x [\vec{A} \cdot \vec{C}] + \hat{j} B_y [\vec{A} \cdot \vec{C}] + \hat{k} B_z [\vec{A} \cdot \vec{C}]$$

$$- \hat{i} C_x [\vec{A} \cdot \vec{B}] - \hat{j} C_y [\vec{A} \cdot \vec{B}] - \hat{k} C_z [\vec{A} \cdot \vec{B}]$$

$$= (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$$