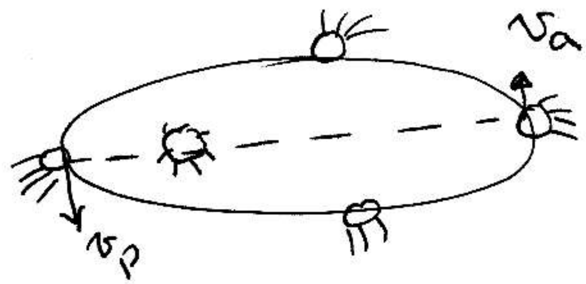
6.16



a) $T = ?$

$$T = \frac{2\pi}{\sqrt{GM_\odot}} a^{3/2}$$

but $GM_\odot = 1.36 \times 10^{20}$

$$a = \frac{r_0}{1-\epsilon} = \frac{(55 \times 10^6 \text{ miles})(1.607 \text{ km/mile})}{1-0.967}$$

$$a = \frac{88.4 \times 10^6 \text{ km}}{0.033} = 2.68 \times 10^9 \text{ km}$$

$$\Rightarrow T = \frac{2\pi}{\sqrt{1.36 \times 10^{20}}} \left(2.68 \times 10^{12}\right)^{3/2}$$

$$\boxed{T = 2.36 \times 10^9 \text{ s} = 75 \text{ Years}}$$

b) using $\ell = r_p v_p$ with $r_p = r_0 = 88.4 \times 10^6$ km

We need $\ell = ?$

6.16 Continues

$$\text{but } \frac{m\ell^2}{k} = \alpha = r_0(1+\epsilon)$$

$$\frac{\cancel{m}\ell^2}{G\cancel{m}M_{sun}} = r_0(1+\epsilon) \Rightarrow$$

$$\ell^2 = (GM_{sun})(r_0)(1+\epsilon)$$

$$= (1.36\times10^{20})(88.4\times10^{9})(1+0.967)$$

$$\ell = 4.86\times10^{15} = \text{angular momentum/mass}$$

$$v_p = \frac{\ell}{r_p} = \frac{\ell}{r_0} = \frac{4.86\times10^{15}}{88.4\times10^{9}} =$$

$$\boxed{v_p = 5.5\times10^{4}\ m/s}$$

using $r_p v_p = r_a v_a \Rightarrow v_a = v_p \frac{r_0}{r_1}$

$$v_a = v_p \frac{1-\epsilon}{1+\epsilon} = (5.5\times10^{4})\frac{0.033}{1.967}$$

$$\boxed{v_a = 9.2\times10^{2}\ m/s}$$

6.18/

$r_0 + r_1 = 2a$

$$l_0 = v_0 r_0 = l_1 = v_1 r_1$$

$$v_1 v_0 = \left(\frac{v_0 r_0}{r_1}\right) v_0 = v_0^2 \frac{r_0}{r_1}$$

$$E = -\frac{k}{2a}$$

$$\downarrow$$

$$\frac{1}{2}\cancel{m} v_0^2 - \frac{G\cancel{m}M}{\cancel{a} r_0} = -\frac{k}{2a} \qquad k = G\cancel{m}M$$

$$V_0^2 = \cancel{2}\, GM\left(\frac{2}{r_0} - \frac{1}{a}\right)$$

$$v_1 v_0 = GM\left(\frac{2}{r_0} - \frac{1}{a}\right)\frac{r_0}{r_1}$$

$$= GM\left(\frac{2}{r_1} - \frac{r_0}{a r_1}\right) = \frac{GM}{r_1}\left(2 - \frac{r_0}{a}\right)$$

$$= \frac{GM}{\cancel{r_1}_{\,1}}\left(\frac{\overset{r_1}{\cancel{2a - r_0}}}{a}\right) = \frac{GM}{a}$$

$$v_1 v_0 = \frac{GM}{a} = \left(\frac{4\pi^2 a^{\cancel{3}\,2}}{T^2}\right)\frac{1}{\cancel{a}}$$

$$= \frac{4\pi^2 a^2}{T^2}$$

6.20

$$V = -\frac{GmM}{r}$$

$$\overline{V} = \frac{1}{T}\int -\frac{GmM}{r}\,dt$$

using $dt = \frac{dt}{d\theta}\,d\theta = \frac{d\theta}{\dot{\theta}} = \frac{d\theta}{l/r^2}$

$$= \frac{r^2 d\theta}{l}$$

$$\Rightarrow \overline{V} = \frac{-1}{T}\int \frac{GmM}{\cancel{r}}\cdot\frac{\cancel{r^2}}{l}\,d\theta = -\frac{1}{T}\frac{GmM}{l}\int r\,d\theta$$

but $r = \frac{\alpha}{1+\epsilon\cos\theta}$

$$\overline{V} = -\frac{1}{T}\frac{GmM}{l}\alpha\underbrace{\int_0^{2\pi}\frac{d\theta}{1+\epsilon\cos\theta}}_{\frac{2\pi}{\sqrt{1-\epsilon^2}}}$$

Substituting $\alpha = a(1-\epsilon^2)$

$l^2 = GM\alpha$

$$\Rightarrow \boxed{\overline{V} = -\frac{k}{a}}$$