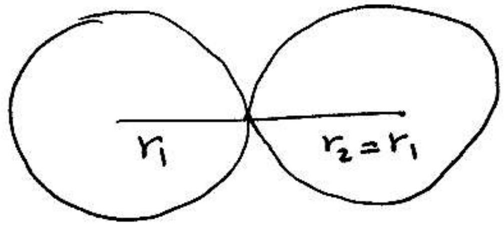
6.1

$m_1 = 1\,kg$ $\qquad$ $m_2 = 1\,kg$



we need r_1 and r_2.

$$m_1 = \rho V = \rho \cdot \tfrac{4}{3}\pi r_1^3$$

ρ: density $\qquad$ V: volume

$$\Rightarrow 1\,kg = \left(11.3 \times 10^3\, kg/m^3\right)\left(\tfrac{4}{3}\right)(\pi)\, r_1^3$$

$$\boxed{r_1 \simeq 0.028\, m = 2.8\, cm}$$

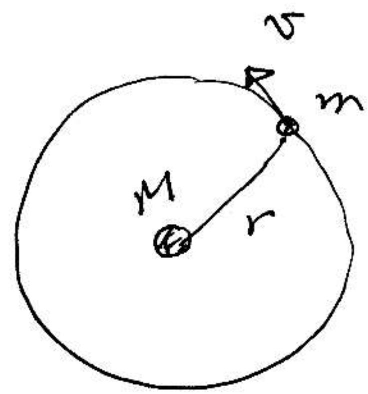
$$F = G\frac{m_1 m_2}{r^2} = \frac{(6.67\times10^{-11})(1)(1)}{(0.028 + 0.028)^2}$$

$$\boxed{F = 2.2\times10^{-9}\, N}$$

Very small (insignificant) force!

$$\Rightarrow F/mg = 2.2\times10^{-8}$$

6.5



$$F = G\frac{mM}{r^2}$$

Using law of motion: $F = ma$ $\downarrow$ $a = \frac{v^2}{r}$

$$\frac{G\cancel{m}M}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2 \quad \text{but } v = \frac{2\pi r}{T}$$

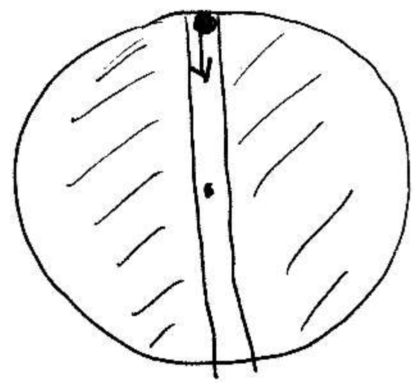
$$\Rightarrow \frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \rightarrow T^2 = \frac{4\pi^2}{GM} r^3$$

$$\boxed{T = \sqrt{\frac{4\pi^2}{GM}}\, r^{3/2}}$$

Which is Kepler's law of areas!

6.3

$$F = -G\frac{mM'}{r'^2}$$



where M' = effective mass of the earth

r' = radius of the effective mass.

$$M' = \rho_{earth} \cdot \tfrac{4}{3}\pi r'^3$$

$$M' = \frac{M_{earth}}{\cancel{\tfrac{4}{3}\pi} R^3} \cdot \cancel{\tfrac{4}{3}\pi} r'^3 = \frac{M}{R^3} r'^3$$

$$\Rightarrow F = -\frac{G m \cdot \frac{M}{R^3} r'^3}{r'^2} = -\frac{GmM}{R^3} r'$$

Defining $k \equiv \frac{GmM}{R^3}$

$F = -kr'$: Simple Harmonic Motion

$$\frac{k}{m} = w^2 = \left(\frac{2\pi}{T}\right)^2 \Rightarrow T^2 = \frac{4\pi^2}{k/m} = \frac{4\pi^2}{GM/R^3}$$

6.3 Continues

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{\cancel{4}\pi^2 \cancel{R^3}}{G \cdot \rho_{earth} \cdot \cancel{4}/3 \pi \cancel{R^3}}$$

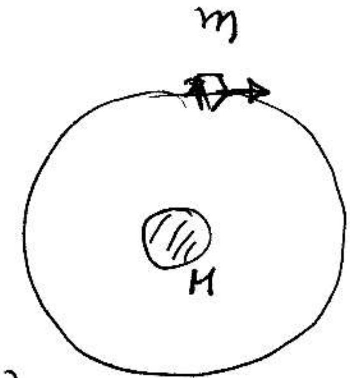
$$\boxed{T = \sqrt{\frac{3\pi}{G\rho_{earth}}}}$$

For $\rho_{earth} \simeq 5.5 \times 10^3 \ kg/m^3$

$$T = \sqrt{\frac{3\pi}{(6.67\times10^{-11})(5.5\times10^3)}}$$

$$\boxed{T = 5067 \ s \simeq 1.4 \ h}$$

6.7



$$F = \frac{GM\cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}}$$

$$\frac{GM}{R} = v^2 = \left(\frac{2\pi R}{T}\right)^2$$

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{\cancel{4}\pi^2 \cancel{R^3}}{G\rho_{earth}\,\cancel{4}/3\,\pi \cancel{R^3}}$$

$$\boxed{T = \sqrt{\frac{3\pi}{G\rho_{earth}}}}$$